NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–I

(Advanced Abstract Algebra)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that the splitting field of $x^4 + 1 = 0$ is $Q(\sqrt{2}, i)$ whose degree over Q is 4.
   (b) Find necessary and sufficient condition on a, b so that the splitting field of irreducible polynomial $x^3 + ax + b$ has degree 3 over Q.
2. (a) State and prove Remainder theorem.
   (b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
3. (a) Prove that every finite extension of a field F is algebraic.
   (b) Prove that if F is a field and K is an extension of F then an element $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.
4. If F is a field of characteristics 0 and a, b are algebraic over F then prove that there exists an element $c \in F[a, b]$ such that $F[a, b] = F[c]$ i.e. $F[a, b]$ is a simple extension.
5. If Ψ be an isomorphism of a field F_1 onto a field F_2 such that $\alpha\Psi = \alpha'$ for every $\alpha \in F_1$ then prove that there is an isomorphism ϕ of $F_1[x]$ on to $F_2[t]$ with the property $\alpha\phi = \alpha\Psi = \alpha'$ for each $\alpha \in F_1$.
6. Let F be a field of characteristics O. Then prove that a polynomial $f(x) \in F(x)$ is solvable by radicals over F if and only if its splitting field K over F has solvable Galois group $G[K, F]$.
7. (a) Show that the Galois group of $x^4 + x^2 + 1$ is the same as that of $x^6 - 1$.
   (b) If K be the splitting field of $x^n - a \in F[x]$. Then show that $G(K, F)$ is a solvable group.
8. (a) Prove that every homomorphic image of a Noetherian (artinian) module is Noetherian (artinian).
   (b) Prove that every finitely generated module is homomorphic image of a finitely generated free module.
9. (a) Prove that the necessary and sufficient condition for a module M to be a direct sum of its two sub modules M_1 and M_2 are that (i) $M = M_1 + M_2$, (ii) $M_1 \cap M_2 = \{0\}$.
   (b) State and prove Schur's theorem.
10. (a) Show that every element in a finite field can be written as the sum of two squares.
    (b) Prove that the group $G[Q(\alpha), Q]$ where $\alpha^5 - 1$ and $\alpha \neq 1$, is isomorphic to cyclic group of order 4.

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EXAMINATION PROGRAMME-2022

M.Sc. Mathematics, Part-I

Date	Papers	Time	Examination Centre
02.12.2022	Paper–I	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
05.12.2022	Paper–II	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
07.12.2022	Paper–III	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
09.12.2022	Paper–IV	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
12.12.2022	Paper–V	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
14.12.2022	Paper–VI	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
16.12.2022	Paper–VII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
20.12.2022	Paper–VIII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–II

(Real Analysis)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Let $f_1, f_2 \in RS(g)$ on [a, b] then prove that $f_1 + f_2 \in RS(g)$ on [a, b] and
$\int_a^b (f_1 + f_2)dg = \int_a^b f_1\, dg + \int_a^b f_2\, dg$.

2. If *f* is defined by $f(x, y) = \dfrac{x^2 y^2}{x^2 + y^2}$, $x^2 + y^2 \neq (0, 0)$
$= 0$ otherwise.
Then show that $f_{xy}(0, 0) = f_{yx}(0, 0)$ although neither f_{xy} nor f_{yx} is continuous at $(0, 0)$. Account for the equality.

3. State and prove Implicit function theorem.

4. (a) Define a function of bounded variation clearly and prove that a bounded monotonic function is a function of bounded variation.
(b) If $f \in BV[a, b]$ and $c \in [a, b]$ then prove that $f \in BV[a, c]$ and $f \in BV[c, b]$ and conversely moreover $V(f, a, b) = V(f, a, c) + V(f, c, b)$.

5. Let *f* be bounded and *g* a non-decreasing function on [a, b]. Then prove that $f \in RS(g)$ if and only if for every $\in > 0$ there exists a partition P of [a, b] such that
$U(P, f, g) - L(P, f, g) < \in$

6. State and prove Schwarz's theorem for a function of two variables.

7. Discuss the continuity and differentiability of the function *f* defined by $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ when $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ at $(0, 0)$.

8. Find the radius of convergence of the following power series.
(i) $\sum_{n=1}^{\infty} \dfrac{\underline{|n}}{n^n} z^n$ (ii) $\sum_{n=1}^{\infty} \dfrac{(\underline{|n})^2}{(2n)!} z^n$

9. If $u_1, u_2, u_3, \ldots\ldots\ldots, u_n$ are functions of $y_1, y_2, \ldots\ldots., y_n$ and $y_1, y_2, y_3 \ldots\ldots., y_n$ are functions of $x_1, x_2, x_3 \ldots\ldots., x_n$ then show that
$\dfrac{\partial(u_1, u_2, \ldots.., u_n)}{\partial(x_1, x_2, \ldots.., x_n)} = \dfrac{\partial(u_1, u_2, \ldots.., u_n)}{\partial(y_1, y_2, \ldots.., y_n)} \cdot \dfrac{\partial(y_1, y_2, \ldots.., y_n)}{\partial(x_1, x_2, \ldots.., x_n)}$.

10. (a) Show that the sequence (f_n) where $f_n(x) = \dfrac{x}{1 + nx^2}$ converges uniformly on R.
(b) Prove that the series $\sum u_n(x)\, v_n(x)$ will be uniformly convergent on [a, b] if
(i) $(v_n(x))$ is a positive monotonic decreasing sequence converging uniformly to zero for $a \leq x \leq b$.
(ii) $|v_n(x)| = \left| \sum_{v=1}^{n} u_r(x) \right| < k$ for every value of *x* in [a, b] and for all integral value of *n* where *k* is a fixed number independent of *x*.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–III
(Measure Theory)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Prove that a necessary and sufficient condition for a Set $S \subseteq R^k$ to be L-measurable is that for every Set $W \subseteq R^k$

$$|W| = |W \cap S| + |W \cap S^c|$$

i.e. $m(W) = m(W \cap S) + m(W \cap S^c)$

(b) If $A_1, A_2, A_3, \ldots\ldots\ldots A_n$ are L-measurable mutually disjoint sets in R^k then show that

(i) $W \subseteq R^k \Rightarrow \left| W \cap \sum_{r=1}^{n} A_r \right| = \sum_{r=1}^{n} |W \cap A_r|$

(ii) $m\left(\sum_{r=1}^{n} A_r\right) = \sum_{r=1}^{n} m(A_r)$

2. Let (A_r) be a sequence of L-measurable sets such that

(i) $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots\ldots\ldots$ and $A = \bigcap_{r=1}^{\infty} A_r$

(ii) $m(A_1) < \infty$

then $m(A) = \underset{r \to \infty}{lt}\; m(A_r)$

3. (a) If A, B are L-measurable sets then show that (i) $A \cup B$ (ii) $A \cap B$ are L-measurable.

(b) Show that the measure of an enumerable set is Zero.

4. State and prove bounded convergence theorem.

5. (a) If f is L-integrable over X then prove that $\left| \int_X f d\mu \right| \leq \int_X |f| d\mu$.

(b) If E and F are measurable sets and f is a integrable function on E + F then show that $\int_{E+F} f d\mu = \int_E f d\mu + \int_F f d\mu$

6. (a) Prove that the class M of L-measurable functions is closed with respect to all arithmetical operations.

(b) Prove that a necessary and sufficient condition for a function f to be L-measurable is that it is the limit of a convergent sequence of simple functions.

7. Use bounded convergence theorem for the function $f_n(x) = \dfrac{nx}{1+n^2x^2}$ to show that whether bounded convergence theorem is true or not in [0, 1].

8. (a) Use Lebesgue dominated convergence theorem to evaluate $\lim_{n \to \infty} \int_0^1 f_n(x)dx$ where $f_n(x) = \dfrac{n^{1/2}x}{1 + n^2x^2}$.

(b) State and prove Lebesgue dominated convergence theorem.

9. Let x be a Lebesgue point of a function f(t) then show that the indefinite integral $F(x) = f(a) + \int_a^x f(t)dt$ is differentiable at each point x and $F'(x) = f(x)$.

10. Prove that every absolutely continuous function is an indefinite integral of its own derivative.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–IV
(Topology)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Let $\{T_i : i \in I\}$ where I is an arbitrary set, be a collection of topologies for X. Then show that the intersection $\cap\{T_i : i \in I\}$ is also a topology for X.
   (b) Define a Topological space, Indiscrete Topology, Discrete topology, co-finite topology and co-countable topology.

2. (a) Define the closure of a Set $A \subseteq X$ where (X, T) is a topological space. If (X, T) is a topological space and A and B are any two subsets of X then prove that,
   (i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (ii) $\overline{A \cap B} = \overline{A} \cap \overline{B}$ (iii) $\overline{\overline{A}} = \overline{A}$
   (b) What do you mean by T_2-space or a Hausdorff space. Prove that every discrete space is a Hausdorff space.

3. (a) Let (X, T_1), (Y, T_2) be two topological spaces. Then prove that a mapping $f : X \to Y$ is $T_1 - T_2$ continuous if and only if for every subset A of X, $f\left(\overline{A}\right) = \overline{f(A)}$.
   (b) Let (X, T_1), (Y, T_2) be two topological spaces. Then prove that a mapping $f : X \to Y$ is closed if and only if $f\left(\overline{A}\right) = \overline{f(A)}\ \forall\ A \subseteq X$.

4. Define a metrizable space and Equivalent metrics giving one suitable example for each.

5. Prove that the union of any family of connected sets having a non-empty intersection is connected.

6. (a) Define a subspace of a topological space (X, T). Let (X, T) be a topological space and $Y \subseteq X$. Then show that the collection $T_Y = \{G \cap Y : G \in T\}$ is a topology on Y.
   (b) What do you mean by Hereditary property of a topological space (X, T). Let (Y, T_Y) be a sub space of (X, T) and let B be a base for T. Then show that $B_Y = \{B \cap Y : B \in \mathrm{B}\}$ is a base for T_Y.

7. (a) Prove that a topological space (X, T) is a T_1-space iff every singleton subset {x} of X is a T-closed set.
   (b) Prove that every second countable space is separable.

8. If (X, T_1), (Y, T_2) are two topological spaces and let $f : X \to Y$ be one-one, onto. Then prove that f is homeomorphism iff $f\left(\overline{A}\right) = \overline{f(A)}\ \forall\ A \subseteq X$.

9. (a) Prove that every closed subset of a compact space is compact.
   (b) Prove that every compact sub space of a Hausdorff space is closed.

10. (a) Prove that every closed-subspace of a normal space is normal.
    (b) Prove that every convergent sequence in a Hausdorff space has unique limit.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–V
(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2022

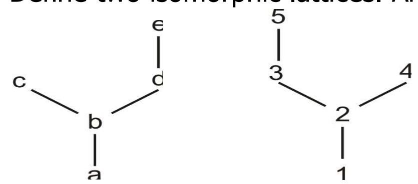
Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Prove that a linear transformation E on a linear subspace L is a projection on some subspace if and only if it is idempotent i.e. $E^2 = E$.

2. Let $T : U \to V$ be a linear transformation. Then prove that
$\dim \cdot \ker(T) + \dim \cdot range(T) = \dim \cdot domain(T)$

3. (a) Define a bounded lattice and show that every finite lattice is bounded.
(b) Define two isomorphic lattices. Are the two lattices shown in the figure isomorphic ?



4. (a) Define a sub lattice with two examples.
(b) Define a Lattice and dual of a statement in a Lattice. Give two examples to make it clear.

5. (a) If L is a complemented lattice with unique complements then the join irreducible elements of L other than O are its atom prove it.
(b) What do you mean by a complemented lattice. Prove that if L be a bounded distributive lattice then complements are unique if they exist.

6. (a) Write the Boolean expression $E(x, y, z)$ first as a sum of products and then in complete sum of products form.
(b) Express $E = Z(x' + y) + y'$ in complete sum of products form.

7. (a) Find the maxiterms of the Boolean algebra $P(A)$ consisting of the subsets of $A = \{a, b, c\}$.
(b) Consider the Boolean algebra D_{210} the divisors of 210. Find the number of sub algebras of D_{210}.

8. (a) Prove that all bases for a vector space V have the same number of vectors.
(b) Show that the set $\{x^2 + 1, 3x - 1, -4x + 1\}$ is linearly independent and the set $\{x + 1, x - 1, -x + 5\}$ is linearly dependent.

9. (a) If W_1 and W_2 are finite dimensional sub spaces of a vector space V, then prove that $W_1 + W_2$ is also finite dimensional and
$\dim \cdot W_1 + \dim \cdot W_2 = \dim \cdot (W_1 \cap W_2) + \dim \cdot (W_1 + W_2)$.
(b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for R^3 and express each of the standard basis vectors as linear combinations of α_1, α_2 *and* α_3.

10. (a) Find the basis for the row space of the following matrix A and determine its rank
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & 1 & 5 \end{bmatrix}.$$
(b) Prove that the row space and the column space of a matrix A have the same dimension.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VI
(Complex Analysis)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |R\, f(z)|^2 = 2|f'(z)|^2$.

   (b) Derive necessary and sufficient condition for $f(z)$ to be analytic in polar co-ordinates.

2. (a) Find the domain of convergence of the series $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5 ....(2n-1)}{n!} \left(\frac{1-z}{2}\right)^n$.

   (b) Examine the behaviour of the power series $\sum_{n=2}^{\infty} \frac{z^n}{n(\log n)^2}$ on the circle of convergence.

3. (a) Show that the transformation $w = \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into a circle of radius unity in w-plane and find the centre of the circle.

   (b) Find the bilinear transformation which maps the points $z = -2, 0, 2$ into points $w = 0, i, -i$ respectively.

4. State and prove Laurent' theorem.

5. State and prove poison's integral formula.

6. State and prove maximum modulus principle.

7. State and prove Cauchy integral formula i.e. If $f(z)$ is analytic within and on a closed contour C and if a is any point within C then $f(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z-a} \cdot dz$.

8. (a) Evaluate the residue of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero.

   (b) Discuss the nature of singularities of the following functions

   (i) $\frac{1}{z(z-1)^2}$ (ii) $\frac{z}{1+z^4}$ (iii) $\frac{1}{z(e^z-1)}$

9. Find the Laurent's series of the function $f(z) = \frac{1}{z^2(1-z)}$ about $z = 0$ and expand $\frac{1}{z^2-3z+2}$ for (i) $0 < |z| < 1$ (ii) $1 < |z| < 2$ (iii) $(z) > 2$.

10. Evaluate any two of the following integrals :–

    (a) $\int_0^{\pi} \frac{a\, d\theta}{a^2 + Sin^2\theta}$, $a > 0$ (b) $\int_0^{\pi} \frac{Cos\, 2\theta}{5 + 4\, Cos\theta} do$

    (c) $\int_0^{\infty} \frac{Cos\, mx}{x^2 + a^2} dx$ (d) $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx$

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VII
(Theory of Differential Equations)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Compute the first three successive approximations for the solution of the Initial Value problem $y^1 = y^2, \quad y(0) = 1$.

2. Define the linear system and show that it satisfies Lipschitz condition and the set of solutions form a vector space.

3. State and prove Cauchy-Peano existence theorem.

4. State and prove Ascolli's Lemma.

5. Show that the function given below satisfy Lipschitz condition in the rectangle indicated and hence find Lipschitz constant $f(x, y) = (y + y^2)\frac{Cos\, x}{2}, \ |y| \le 1, |x - 1| \le ½$.

6. Find the nature of the critical point (0, 0) of the system of equations $\frac{dx}{dt} = 3x + 4y$ and $\frac{dy}{dt} = 3x + 2y$.

7. Test the stability of the non-linear system $\frac{dx}{dt} = x + 4y - x^2, \frac{dy}{dt} = 6x - y + 2xy$. Further make a comment on stability.

8. (a) Describe orthogonal property of Laguerre polynomial.
   (b) What is the meaning of generating function for Legendre polynomial ? Hence find it.

9. Find the series solution of Bessel's differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0$.

10. (a) Solve the differential equations by matrix method

$$\frac{dx_1}{dt} = 9x_1 - 8x_2$$

$$\frac{dx_2}{dt} = 24x_1 - 9x_2$$

   The initial conditions for which are $x_1(0) = 1, \ x_2(0) = 0$.

   (b) Define fundamental matrix and show that a necessary and sufficient condition that a solution matrix G to be fundamental matrix is $G(x) \ne 0$ for $x \in I$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VIII
(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) If α, β, λ are any three Cardinal numbers then prove that
   (i) $\alpha + (\beta + \lambda) = (\alpha + \beta) + \lambda$ (ii) $\alpha(\beta + \lambda) = \alpha\beta + \alpha\lambda$
   (b) Prove that every set can be well ordered.
2. (a) If A and B are countable sets then show that A × B is also countable.
   (b) Prove that the set of all real numbers R is uncountable.
3. (a) What is Axiom of choice ? Show that the Axiom of choice is equivalent to Zermelo's postulates.
   (b) Prove that Zorn's lemma implies well ordering theorem.
4. (a) If a_n is the nth term of the Fibonaced sequence and $\alpha = \frac{1 + \sqrt{5}}{2}$ then $a_n > \alpha^{n-1} \ \forall \ n > 1$.
   (b) Factorize 493 by Euler's Factorization method.
5. (a) State and prove division algorithm in theory of numbers.
   (b) Find g.c.d. of 28 and 49 and express it as a linear combination of 28 and 49.
6. (a) Solve the congruence $x^3 \equiv 5$ (mod 13).
   (b) Show that $T(n) = \begin{bmatrix} n \\ 1 \end{bmatrix} + \begin{bmatrix} n \\ 2 \end{bmatrix} + \begin{bmatrix} n \\ 3 \end{bmatrix} + \ldots\ldots.. + \begin{bmatrix} n \\ n \end{bmatrix}$.
7. Show that for a geodesic $T^2 = (K - K_1)(K - K_2)$ where K is curvature and T is torsion.
8. Define an Umbilic. Prove that in general three lines of curvature pass through an umbilic.
9. (a) Prove that an undirected graph is a tree iff there is a unique path between any two vertices.
   (b) If a tree has n vertices of degree 4 then find the value of n.
10. (a) Show that a complete graph of n vertices is a planner if $n \leq 6$.
    (b) If a tree has n vertices of degree 1, two vertices of degree 2, four vertices of degree 4 then find the value of n.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
Calculator is Allowed.

1. Prove that the divided differences can be expressed as the product of multiple integrals.
2. (a) Show that $\Delta^n x^{(n)} = \lfloor n \, h^n$ and $\Delta^{n+1} x^{(n)} = 0$.

   (b) Express $2x^3 - 3x^2 + 3x - 10$ and its difference in factorial notation, the interval of differencing being unity.
3. Find $f'(7.50)$ from the following table.

x	7.47	7.48	7.49	7.50	7.51	7.52	7.53
$y = f(x)$	.193	.195	.198	.201	.203	.206	.208

4. (a) Use the method of separation of symbols to prove that
   $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \text{.........} + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n-1}$.

   (b) Obtain the estimate of the missing numbers in the following table.

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	?	64	?	216	343	512

5. (a) Solve the equation $y_{h+2} - 4y_{h+1} + 4yh = 0$.

   (b) Find the sum to n terms of the series whose x^{th} term is $2^x (x^3 + x)$.
6. Find the polynomial of fifth degree from the following data.
   $u_0 = -18,\ u_1 = 0,\ u_3 = 0,\ u_5 = -248,\ u_6 = 0,\ u_9 = 13140$
7. Find the value of $\int_0^1 \frac{dx}{1 + x^2}$ by using Simpson's $\frac{1}{3}^{\text{rd}}$ and $\frac{3}{8}^{\text{th}}$ rule, hence obtain the approximate value of π in each case.
8. (a) By using the method of iteration find a real root of $2x - \log_{10}^{x} = 7$.

   (b) Find the root of the equation $x^3 - 6x - 11 = 0$ which lies between 3 and 4.
9. (a) Use synthetic division to solve $f(x) = x^2 - 1.0001x + 0\cdot 9999 = 0$ in the neighbourhood of x = 1.

   (b) Solve the equation $3x - Cosx - 1 = 0$ by false position method and Newton Raphson method.
10. (a) Obtain an approximation in the sense of the principle of least squares in the form of polynomial of second degree to the function $f(x) = \frac{1}{1 + x^2}$ in the range $-1 \le x \le 1$.

    (b) Fit a second degree parabola to the following :–

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

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EXAMINATION PROGRAMME-2022
M.Sc. Mathematics, Part-II

Date	Papers	Time	Examination Centre
27.01.2023	Paper–IX	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
31.01.2023	Paper–X	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
02.02.2023	Paper–XI	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
04.02.2023	Paper–XII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
06.02.2023	Paper–XIII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
08.02.2023	Paper–XIV	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
10.02.2023	Paper–XV	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
14.02.2023	Paper–XVI	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna

NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–X

(Functional Analysis)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) If x and y are any two vectors in an inner product space then prove that $|(x, y)| \leq \| x \| \cdot \| y \|$.

   (b) Show that the inner product space is jointly continuous.

2. State and prove Riesz Lemma.

3. State and prove closed graph theorem.

4. (a) Show that a normed linear space is a metric space under the property $\big| \| x \| - \| y \| \big| \leq \| x \| - \| y \|$.

   (b) If x and y are any two vectors on a Hilbert space H then show that $\| x - y \|^2 + \| x + y \|^2 = 2\left(\| x \|^2 + \| y \|^2 \right)$.

5. Consider a real number p such that $1 \leq p < \infty$. Denote l_p the space of all sequences $x = (x_1, x_2, x_3, \ldots\ldots\ldots\ldots)$ of scalars such that $\sum_{n=1}^{\infty} | x_n | p < \infty$ with norm defined by $\| x \|_p = \left(\sum_{n=1}^{\infty} | x_n | p \right)^{1/p}$. Show that l_p is a Banach space.

6. If N and N' be normal linear spaces and let T : N $\rightarrow$ N' be any linear map. If N is finite dimensional then prove that T is continuous or bounded.

7. If f and g belong to $L^p(a, b)$ where p > 1 then prove that $\| f + g \|_p \leq \| f \|_p + \| g \|_p$.

8. (a) Prove that if M and N are closed linear sub spaces of a Hilbert space H $s \cdot t$ $M \perp N$, then M + N is also closed.

   (b) If T is a normal operator on a Hilbert space H then prove that $\| T^2 \| = \| T \|^2$.

9. If N and N' be normal linear spaces and T be a linear transformation of N into N'. Then show that the following conditions are equivalent to one another.

   (i) T is continuous.

   (ii) T is continuous at the origin (i.e. $x_n \rightarrow 0 \Rightarrow T(x_n) \rightarrow 0$)

   (iii) T is bounded (i.e. $\exists$ a real number $K \geq 0$ $s \cdot t$ $\| T(x) \| \leq K \| x \| \; \forall \; x \in N$).

   (iv) If $S = \{ x : \| x \| = 1 \}$ is closed sphere in N then the image T(S) is a bounded set in N'.

10. State and prove open mapping theorem.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XI
(Partial Differential Equations)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) Solve $pt - qs = q^3$, by Monge's method.

   (b) Solve $(q + 1)s = (p + 1)t$ by Monge's method.

2. Solve

   (a) $(D^2 + 3DD' + 2D'^2)z = x + y$

   (b) $(D^2 + 2DD' + D'^2)z = e^{3x + 2y}$

3. (a) Find the complete integral of $(p^2 + q^2)x = pz$. (use Charpit's method)

   (b) Solve $p^2 x + q^2 y = z$ by Jaicobi's method.

4. (a) Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x = 1,\ y = 0,\ z = 1$.

   (b) Find the characteristics curve of $2yu_x + (2x + y^2)u_y = 0$ passing through (0, 0).

5. Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its general solution.

6. Using the method of separation of variables solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.

7. Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\left(\frac{\partial u}{\partial t}\right)$ satisfying the conditions $0 = u(0, t) = u(l, t),\ u(x, 0) = (x - x^2)$.

8. Show that the general solution of the wave equation $c^2\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial^2 u}{\partial t^2}$ is $u(x, t) = \phi(x + ct) + \psi(x - ct)$ where ϕ and ψ are arbitrary functions.

9. Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(x, 0) = u(x, b) = 0$ for $0 < x < a$, and $u(a, y) = f(y)$ for $0 < y < 6$.

10. Transform the Laplaces equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ in the cylindrical co-ordinates.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XII

(Analytical Dynamics)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Determine the kinetic energy and the moment of momentum of a rigid body rotating about a fixed axis.

2. (a) Prove that in a simple dynamic system $T + V =$ constant where T and V have their usual meaning.
   (b) Derive the formula for the Kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of Kinetic energy.

3. (a) Explain the principle of least action and hence establish it in terms of arc length of a particle path.
   (b) A particle moves in a plane under a central force depending on its distance from the origin. Then construct the Hamiltonian of the system and derive Hamilton's equation of motion.

4. State and prove Jaicobi-Poisson theorem.

5. Derive Euler's equation of motion for the motion of rigid body about a fixed point.

6. (a) State and prove Jaicobi-theorem.
   (b) A particle of mass m moves in a force field whose potential in spherical co-ordinates is given by $V = \frac{\lambda \, Cos\, \theta}{r^2}$, then write down the Hamilton Jaicobi equation and derive the complete solution.

7. (a) Show that the transformation $Q = q \tan p$, $P = \log(Sin\, p)$ is canonical.
   (b) Define the generating function of a transformation and give an example of a generating function of transformation.

8. (a) A bead is sliding on a uniformly rotating wire in a force free space. Derive its equation of motion.
   (b) Derive Lagrange's equation of impulsive motion in a Holonomic dynamical system.

9. (a) Construct Routhian function and Routh's equation for the solution of a problem involving cyclic and non-cyclic co-ordinates.
   (b) Use Routhian equation of motion to determine the motion of a uniform heavy rod turning about one end which is fixed.

10. (a) Explain the difference between possible displacement and virtual displacement. Give one example of each.
    (b) Explain the terms (i) degree of freedom, (ii) Constraints, (iii) generalized co-ordinates and classify the dynamical systems based on different types of constraints.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIII
(Fluid Mechanics)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Derive Cauchy-Riemann differential equation in polar form.
2. State and prove Kelvin's circulation theorem.
3. Obtain the equation of continuity in spherical polar co-ordinates.
4. Derive Euler's equation of Fluid motion.
5. (a) Show that $u = Axy$, $v = A(a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Determine the stream function of fluid motion.
   (b) What do you mean by source and sink ? Find the complex potential due to a source of strength m placed at the origin.
6. (a) A two dimensional flow field is given by $\psi = xy$, then (i) show that the flow is irrotational (ii) Find the velocity potential (iii) find the stream lines and potential lines.
   (b) Show that the two dimensional irrotational motion, stream function satisfies Laplace's equation.
7. Obtain the boundary layer equations in two dimensional flow.
8. Derive the equation of energy for an incompressible fluid motion with constant fluid properties.
9. Show that the vorticity vector Ω of an incompressible viscous fluid moving under no external forces satisfies the differential equation.

   $\frac{D\Omega}{Dt} = (\Omega \bullet \nabla) q + \nu \nabla^2 \Omega$ Where ν is the kinematic of viscosity.
10. (a) Find the principal stresses and principal directions of stress at a point (1, 1, 1) if the components of the stess tensor are given by

$$\sigma y = \begin{pmatrix} 0 & 2y_3 & 0 \\ 2y_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

   (b) What type of motion do the following velocity components constitute ?
   $u = a + by - cz$, $v = d - bx + ez$, $w = f + cx - ey$ where a, b, c, d, e, f are arbitrary constants.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XIV

(Operation Research)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. (a) By using Lagrange's multiplier method solve the NLPP
$z = a x_1^2 + b x_2^2 + c x_3^2$ where $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 1$.
(b) Obtain the feasible solution of non-linear programming
Min $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
Subject to $x_2 \le 8$, $x_1 + x_2 \le 10$, $x_1 \ge 0$, $x_2 \ge 0$.

2. Explain in details the dual simplex method elaborating each step.

3. Using simplex method solve L.P.P.
Max $z = 4x_1 + 10x_2$
Subject to the condition $2x_1 + x_2 \le 50$, $2x_1 + 5x_2 \le 10$, $2x_1 + 3x_2 \le 90$; $x_1 \ge 0$, $x_2 \ge 0$.

4. (a) Reduce feasible solution $x_1 = 2$, $x_2 = 4$, $x_3 = 1$ of the system $2x_1 - x_2 + 2x_3 = 2$ and $x_1 + 4x_2 = 18$ to a basic feasible solution and mention its kind (degenerate or non-degenerate).
(b) Find basic feasible solution of the system $2x_1 + x_2 + 4x_3 = 11$, $3x_1 + x_2 + 5x_3 = 14$.

5. (a) Solve the game problem whose pay off matrix is $\begin{bmatrix} 6 & 2 & 7 \\ 1 & 9 & 3 \end{bmatrix}$.
(b) Describe the method of constructing the solution of 'Game Problem' where the game is without saddle point.

6. Apply two phase method to compute the solution of
Min $z = x_1 + x_2$
Subject to $2x_1 + x_2 \ge 4$, $x_1 + 7x_2 \ge 7$; $x_1 \ge 0$, $x_2 \ge 0$.

7. (a) Prove that the intersection of any finite number of convex sets is a convex set.
(b) Define hyper plane and hyper sphere. Prove that every hyper plane in R^n is a convex set.

8. Find the dual of the following L.P.P.
Min $z = x_1 + x_2 + x_3$
S.t. $x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \le 3$, $2x_2 - x_3 \ge 4$; $x_1, x_2 \ge 0$; x_3 is unrestricted in sign.

9. Solve the following assignment problem.

		Man				
		I	II	III	IV	V
	A	1	3	2	3	6
	B	2	4	3	1	5
Task	C	5	6	3	4	6
	D	3	1	4	2	2
	E	1	5	6	5	4

10. Solve the following transportation problem.

	To			Supply
	1	2	3	
From 1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XV
(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Find Fourier transform of $F(x)=\begin{cases}1, & |x|<a\\ 0, & |x|>a\end{cases}$ and hence prove that $\int_0^\infty \frac{Sin^2 ax}{x^2}dx = \frac{\pi a}{2}$.
   (b) State and prove convolution theorem of Fourier transforms.
2. (a) Find the Fourier sine transform of $x e^{-\frac{x^2}{2}}$.
   (b) Find the Fourier cosine transform of e^{-x^2}.
3. Find (a) $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$ (b) $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$
4. Find the Laplace transform of
   (a) $L\left\{Sin\sqrt{t}\right\}$ (b) $L\left\{Sin^2 at\right\}$ (c) $L\left\{e^{at}\ Cos\, bt\right\}$ (d) $L\left\{e^{-t}\ Cos^2 t\right\}$
5. Using Laplace transform solve the following differential equations
   (a) $(D+1)^2 y = t$, given that $y=-3$, when t = 0 and y = -1 when t = 1.
   (b) Solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=2\,Sin\,t$, given that $y=\frac{dy}{dt}=0$ when t = 0.
6. The maintenance and re-sale value per year of a machine whose purchase prices is Rs. 7000/- is given below :—

Year	1	2	3	4	5	6	7	8
Maintenance Cost in Rs.	900	1200	1600	2100	2800	3700	4700	5900
Re-sale Value in Rs.	4000	2000	1200	600	500	400	400	400

   When should the Machine be replaced.
7. (a) Describe Firedholm integral equation and voltera integral equation.
   (b) Explain about the Fredholm integral equations of three kinds.
8. (a) Show that any linear combination of tensors of type (r, s) is a tensor of type (r, s).
   (b) Prove that a skew symmetric tensor of rank two has $\frac{N}{2}(N-1)$ independent components.
9. (a) Show that $\left\{\begin{matrix} i \\ i\ j\end{matrix}\right\} = \frac{\partial \log\left(\sqrt{g}\right)}{\partial x^j} = \frac{\partial\left(\log\sqrt{-g}\right)}{\partial x^j}$.
   (b) What do you mean by Christoffel's symbols and prove that
   $[ij,\,k]+[jk,\,i]=\frac{\partial g_{ik}}{\partial x^j}=\partial_j g_{ik}$.
10. (a) Prove that the inner product of tensors A^i_{ij} and B^{ij}_i is a tensor of rank three.
    (b) Define the inner and outer product of two tensors and prove that the outer product of two tensors is a tensor of rank equal to the sum of ranks of the two tensors.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XVI

(Programming in 'C')

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. What is an Operator ? Describe different types of operators in C with examples.
2. What is control statement is C Language ? Explain with the help of an example.
3. What is meant by looping in C ? Explain some of the looping statements with examples.
4. What is function ? Are functions required when writing a C program ?
5. Write a programme in C to find the roots of a quadratic equation.
6. What are reserved words ? What is the difference between the expression "++a" and "a++" ? Explain with examples.
7. Write a program in C to check whether a given number is a Prime Number.
8. Discuss the features of C programming language in detail. What is difference between compiler & interpreter ?
9. What is debugging ? When is the "void" keyword used in a function ?
10. Write short notes on any two of the following :—
    (i) Array
    (ii) Global variables
    (iii) Break and continue statement
    (iv) GOTO statement

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M.Sc. Mathematics, Part–II, Paper–XVI

Practical Counselling & Examination Programme, 2022

A. Counselling Class Programme - ONLINE

अनुक्रमांक	*तिथि*	*समय*	*स्थान*
All New & Old Students	22.02.2023 एवं 25.02.2023	3.00 PM to 6.00 PM	**ONLINE** (Online Counselling Class, Microsoft Teams Platform पर कराया जायेगा, इसलिए वे Link से जुड़ने से पहले Play Store से Microsoft Teams Install कर लें, तदुपरान्त Name और Enrollment No. दर्ज कर, ससमय परामर्श कक्षा में शामिल होवें । Counselling Class का Link नालन्दा खुला विश्वविद्यालय के वेबसाईट www.nou.ac.in के Student Corner Section में दिया गया है ।)

B. Practical Examination Programme - OFFLINE

अनुक्रमांक	*परीक्षा की तिथि*	*परीक्षा का समय*	*परीक्षा का स्थान*
200290001 to 200290269	27.02.2023	11.30 AM to 1.30 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
200290270 to 200290499	27.02.2023	2.30 PM to 4.30 PM	
200290500 to 200290925	28.02.2023	11.30 AM to 1.30 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
200290926 to 200291285 190290001 to 190290107	28.02.2023	2.30 PM to 4.30 PM	
190290108 to 190290599	01.03.2023	11.30 AM to 1.30 PM	School of Computer Education & IT Nalanda Open University, 12th Floor, Biscomaun Tower, Patna-800001
190290600 to 190291084 180290001 to 180291000 170290001 to 170291000	01.03.2023	2.30 PM to 4.30 PM	

इस कार्यक्रम में किसी भी परिस्थिति में परिवर्तन नहीं होगा ।