NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-I

PAPER–I (Honours)

(Set Theory, Matrices, Abstract Algebra, Theory of Equations and Trigonometry)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Define the Cartesian product of two non-empty sets A and B and prove that if A, B, C are any three non--empty sets then
$A \times (B - C) = A \times B - A \times C$ and
$A \times (B \cap C) = (A \times B) \cap (A \times C)$

(b) Define an equivalence relation on a non-empty set A and if R_1 and R_2 are any two equivalence relations on A then show that $R_1 \cap R_2$ is also an equivalence relation on A.

2. (a) What do you mean by a Lattice and a complete Lattice, Give one example of each.

(b) What do you mean by a partially ordered set. If X is any non-empty set then show that $(P(X), \subseteq)$ is a partially ordered set.

3. (a) If $f: X \to Y$ and $A \subseteq X$, $B \subseteq X$ then show that $f(A \cap B) \subseteq f(A) \cap f(B)$.

(b) What do you mean by a denumerable set. Prove that every infinite set has a denumerable subset.

4. (a) Show that a countable union of countable sets is countable.

(b) Show that the set R of all real numbers is uncountable.

GROUP 'B'

5. Find the rank of the matrix $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix}$.

6. Solve by matrix method the following simultaneous equations.
$x + y + z = 6, \quad 2x + y - 3z = -5, \quad 3x - 2y + z = 2$

7. (a) Prove that $A(\text{adj } A) = (\text{adj } A)A = |A|$ where A is n-rowed square matrix.

(b) Find the eigen values of the matrix. $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$.

8. Show that $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I = 0$. Hence or otherwise find the inverse of A.

GROUP 'C'

9. (a) Find the condition that the cubic $x^3 - px^2 + qx + r = 0$ should have its roots be in Harmonic progression.

(b) The equation $3x^4 - 25x^3 + 50x^2 - 50x + 12 = 0$ has two roots whose product is $2i$, find all the roots.

10. (a) Find the expansion of $\sin\theta$ is ascending powers of θ.

(b) State and prove Gregories series for expansion of $\tan^{-1}x$ in ascending powers of x.

11. (a) Prove that if for every element 'a' in a group G, $a^2 = e$ then G is an abelian group.

(b) Prove that any two left or right cosests of a subgroup of a group G are either disjoint or identical.

12. (a) State and prove Lagranges Theorem.

(b) Prove that the intersection of two subgroups of a group G is also a subgroup of that group.

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NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-I
PAPER–II (Honours)

(Differential Calculus, Integral Calculus and Analytical Geometry of Three Dimensions)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) State and prove Maclaurin's theorem.

   (b) Prove that $\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \dots\dots\infty$

2. (a) If $v = \tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \sin 2v$.

   (b) State and prove Euler's theorem for Homogeneous function of two independent variables x and y of degree n.

3. Evaluate the following limits.

   (a) $\underset{x\to 0}{Lt}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$ (b) $\underset{x\to\frac{\pi}{2}}{Lt}(\sin x)^{\tan x}$.

4. (a) Show that the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches the curve $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$.

   (b) Find the pedal equation of the curve $r^n = a^n \sin(n\theta)$.

5. (a) State and prove Leibnitz's theorem.

   (b) If $y = \sin(m \sin^{-1} x)$ then prove that :
   $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$

GROUP 'B'

6. (a) If $I_{m,n} = \int \cos^m x \sin^n x\, dx$ then show that $(m+n)\, I_{m,n} = \cos^{m-1} x \cdot \sin^{n+1} x + (m-1)I_{m-2,n}$

   (b) Evaluate $\underset{r\to\infty}{lt}\sum_{r=1}^{n}\frac{r^3}{r^4+n^4}$.

7. Evaluate any ***Two*** of the following :—

   (a) $\int\frac{d\theta}{(a^2\sin^2\theta + b^2\cos^2\theta)^2}$ (b) $\int(\sqrt{\tan x} + \sqrt{\cot x})dx$ (c) $\int\sqrt{\sec x + 1}\, dx$

8. Evaluate

   (a) $\int_0^1 \frac{\log(1+x)}{(1+x^2)}$ (b) $\int_0^\pi x\log(\sin x)dx$

9. Find the area between the curve $x(x^2 + y^2) = a(x^2 - y^2)$ and its asymptote. Also find the area of the loop.
10. Find the volume formed by the revolution of the loop of the curve $y^2(a-x) = x^2(a+x)$.

GROUP 'C'

11. (a) Find the angle between two lines whose direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) respectably

    (b) Find the equation of the plane cutting off intercepts a, b, c from the axes.
12. Show that $3x^2 + 4y^2 + 5z^2 - 6yz - 4zx - 2xy = 0$ represents a pair of planes.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-I, PAPER–I (Subsidiary)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer any **Eight** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) If a and b are any two elements of a group G, then prove that the equation $ax = b$ and $ya = b$ have unique solution in G.
   (b) If G is group then prove that $(ab)^{-1} = b^{-1}a^{-1}\ \forall\ a, b \in$ G.
2. Prove that the set Pn of all permutations on n symbols is a finite non-abelian group of order n with respect to composition of mappings as the operation.
3. What do you mean by an equivalence relation on a set A. If R_1 and R_2 are two equivalence relations on A then show that $R_1 \cap R_2$ is also an equivalence relation on R.
4. (a) Define Reflexive, Symmetric and Transitive relations giving one example of each.
   (b) Define the Cartesian product of two non-empty sets A and B. If A, B, C are three non-empty sets then prove that $(A - B) \times C = A \times C - B \times C$.

GROUP 'B'

5. Prove that the sequence whose n^{th} term is $\left(\sqrt{n+1} - \sqrt{n}\right)$ is convergent.
6. Prove that a monotonic increasing sequence which is bounded above is convergent.
7. Prove that every convergent sequence is bounded.
8. Show that the sequence (x_n) where
   $x_1 = 1, x_n = \sqrt{2 + x_{n-1}}$ is convergent and it converges to 2.
9. Find $(1+i)^{\frac{1}{3}}$?
10. Reduce $(\alpha + i\beta)^{x+iy}$ in the form of $A + iB$.
11. State and prove De-Moivre's theorem.

GROUP 'C'

12. Evaluate:
    (a) $\underset{x \to 0}{Lt} (\cot x)^{\left(\frac{1}{\log x}\right)}$ (b) $\underset{x \to 0}{Lt} \dfrac{x \cos x - \log(1+x)}{x^2}$
13. If $y = \tan^{-1} x$ then prove that:
    $(1 + x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.$
14. Apply Maclaurin's theorem to expand $e^{x \sec x}$ as far as the term containing x^3.
15. (a) Give the geometrical meaning of scalar triple product of three vectors $\vec{a}, \vec{b}, \vec{c}$.
    (b) Prove that : $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$.
16. (a) Prove that the two spheres:
    $S_1 \equiv x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and
    $S_2 \equiv x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$
    cut each other orthogonally if $2(u_1u_2 + v_1v_2 + w_1w_2) = d_1 + d_2$.
    (b) If the point $(at_1^2, 2at_1)$ is one extremity of a focal chord of the parabola $y^2 = 4ax$ then find the co-ordinates of the other extremity and hence show that the length of the chord is $a\left(t_1 + \dfrac{1}{t_1}\right)^2$.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-II

PAPER–III (Honours)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Prove that every compact subset of R is closed.
   (b) Prove that every closed subset of a compact set in R is compact.

2. (a) Prove that Int. (A) is an open set.
   (b) Prove that a set E in R is compact if and only if E is closed and bounded.

3. (a) State and prove Bolzano Weierstrass theorem.
   (b) State and prove Heine-Borel theorem.

GROUP 'B'

4. (a) Prove that every Cauchy Sequence of real numbers is convergent.
   (b) If (x_n) is a sequence where $x_n = \left(\sqrt{n+1}-\sqrt{n}\right)$ for all $n \in N$, then show that it is convergent and find its limit.

5. (a) Prove that every bounded monotonically increasing sequence converges to its least upper bound.
   (b) Prove that every monotonically decreasing sequence which is bounded tends to its greatest lower bound.

6. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$.
   (b) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$.

7. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{\cos nx}{n}\right)$.
   (b) State and prove Logarithmic ratio test.

GROUP 'C'

8. If W_1, W_2 are two sub spaces of a finite dimensional vector space V over a field F then show that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$.

9. (a) Prove that the set $(1, i, 0), (2i, 1, 1), (1, 1 + i, 1 - i)$ is a basis for $V_3(C)$.
   (b) Define the eigen values and eigen vectors of a square matrix and compute the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

10. Prove that $T : V_2(R) \to V_3(R)$ defined by $T(a, b) = ((a + b), (a - b), b)$ is a linear transformation.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-II

PAPER–IV (Honours)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Obtain the primitive and singular solution of the equation $xp^2 - 2yp + 4x = 0$.
   (b) Solve the differential equation $(8p^3 - 27)x = 12p^2y$ and investigate whether a singular solution exists.
2. (a) Solve the differential equation by the method of variation of parameters $\frac{d^2y}{dx^2} + a^2y = \operatorname{cosec} ax$.
   (b) Solve : $\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + (4\operatorname{cosec}^2 x)y = 0$
3. (a) Solve : $y = (1+p)x + ap^2$
   (b) $p(p + x) = y(x + y)$.

GROUP 'B'

4. (a) Prove that : $[\vec{a}\ \vec{b}\ \vec{c}][\vec{p}\ \vec{q}\ \vec{r}] = \begin{bmatrix} \vec{a}\cdot\vec{p} & \vec{b}\cdot\vec{p} & \vec{c}\cdot\vec{p} \\ \vec{a}\cdot\vec{q} & \vec{b}\cdot\vec{q} & \vec{c}\cdot\vec{q} \\ \vec{a}\cdot\vec{r} & \vec{b}\cdot\vec{r} & \vec{c}\cdot\vec{r} \end{bmatrix}$.
   (b) Find the volume of the parallelopiped whose edges are represented by :
   $\vec{i}+\vec{j}+\vec{k},\ \vec{i}-\vec{j}+\vec{k},\ \vec{i}+2\vec{j}-\vec{k}$.
5. (a) Prove that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\ \ \vec{b}\ \ \vec{c}]^2$.
   (b) Evaluate : $\frac{d^2}{dt^2}\left\{\left(\vec{r}\times\frac{d\vec{r}}{dt}\right)\times\frac{d^2\vec{r}}{dt^2}\right\}$.
6. (a) Find the unit normal vector to the level surface $x^2 + y - z = 4$ at the point (2, 0, 0).
   (a) If $\vec{a}$ and $\vec{b}$ are constant vectors and $\vec{r} = (x, y, z)$, then prove that :
   $\nabla\cdot\left\{\vec{a}\times\left(\nabla\left(\frac{1}{\vec{r}}\right)\right)\right\} = 0$.
7. (a) Prove that $\nabla\cdot(\nabla\times\vec{u}) = 0$ or div. curl $\vec{u} = 0$.
   (b) Prove the $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot(\nabla\times\vec{u}) - \vec{u}\cdot(\nabla\times\vec{v})$.

GROUP 'C'

8. In a simple Harmonic motion if u, v, w be the velocities at distances a, b, c respectively from a fixed point on the straight line which is not the centre of the force, then Show that the periodic time is given by the equation:
   $$\frac{4\pi^2}{T}(a-b)(b-c)(c-a) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$
9. State and prove the necessary and sufficient condition for the principle of virtual work.
10. (a) What are the forces that can be neglected during forming the equation of virtual work.
    (b) Show that the modulus of an elastic string is equal to the force which would stretch a light string to twice its natural length.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-II

PAPER–II (Subsidiary)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Eight** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP-A

1. Find the area of the loop of the curve $x^3 + y^3 = 3axy$.

2. Show that the length of the loop of the curve $3ay^2 = x (x - a)^2$ is $\frac{4a}{\sqrt{3}}$.

3. Find the area of the surface of revolution formed by revolving the loop of the curve $9ay^2 = x (3a - x)^2$ about the x-axis.

4. Find the volume of the solid generated by the revolution of the upper half of the loop of the curve $y^2 = x^2 (2 - x)$.

5. Find the perimeter of the loop of the curve $9ay^2 = (x - 2a) (x - 5a)^2$.

6. Evaluate any ***Two*** of the following integrals :—

   (a) $\int \frac{dx}{\sqrt{(x-a)(x-b)}}$ (b) $\int \frac{2x+3}{\sqrt{x^2+x+1}}dx$ (c) $\int \frac{x^2 dx}{(1-x^4)\sqrt{1+x^4}}$

7. Evaluate any ***Two*** of the following :—

   (a) $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$ (b) $\int_0^{\pi} \frac{dx}{a+b\cos x}$ (c) $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$

8. (a) Evaluate $\lim\limits_{n\to\infty}\left[\frac{n}{n^2} + \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots\dots\dots + \frac{n}{n^2+(n-1)^2}\right]$.

   (b) Obtain a reduction formula for $\int \sin^m x \cos^n x\, dx$.

9. Solve the following differential equations :—

   (a) $p (p + x) = y (x + y)$ (b) $y = x\left\{\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2\right\}$

10. Solve the following differential equations :—

   (a) $\frac{d^2y}{dx^2} + a^2 y = \sec ax$ (b) $\frac{d^2y}{dx^2} - y = x \sin x$

GROUP 'B'

11. (a) Define a convex set $S \subseteq R^2$ and prove that the sphere is a convex set.

    (b) Prove that a hyper plane is a closed set.

12. Find the equation of the sphere which passes through the point (α, β, γ) and the circle $x^2 + y^2 + z^2 = a^2,\ z = 0$.

13. Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9,\ x - y + z = 3$.

GROUP 'C'

14. Define simple Harmonic Motion and show that how two simple Harmonic motions can be compounded in a straight line.

15. If forces P, Q, R act along the lines x = 0, y = 0 and $x\,Cos\alpha + y\,Sin\alpha = p$. Find the magnitude of the resultant and its line of action.

16. Find the equation of line of action of co-planar forces and its resultant.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–V (Honours)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Define a Cauchy sequence in a metric space (x, d) and prove that every convergent sequences in (x, d) is a Cauchy sequence in (x, d).

   (b) Define the convergence of a sequence (x_n) in a metric space (x, d) and prove that limit of sequence in (x, d) if it exists is unique.

2. (a) State and prove Cauchy Schwartz inequality.

   (b) State and prove Minkowsky's inequality.

3. Prove that (R^n, d) is complete where d on R^n is defined as d(x, y) = $\left[\sum_{i=1}^{n} |x_i - y_i|^2\right]^{\frac{1}{2}}$.

4. (a) In a metric space (x, d) prove that any finite intersection of open sets in X is open.

   (b) In a metric space (x, d) prove that the union of an arbitrary collection of open sets is open.

5. (a) If M and N are two subsets of a metric space (x, d) then show that $\overline{M U N} = \overline{M} \cup \overline{N}$.

   (b) Let (x, d) be a metric space and $A \subseteq X$ then show that A is closed if and only if $A \subseteq \overline{A}$.

GROUP 'B'

6. Let (X, T) be a Topological space and A and B are any two subsets of X and $\overline{A}$ denotes the closure of A then prove that :

   (a) $\overline{\phi} = \phi$ (b) $A \subseteq \overline{A}$ (c) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$

   (d) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (e) $(\overline{A \cap B}) \subseteq \overline{A} \cap \overline{B}$ (f) $\overline{\overline{A}} = A$

7. (a) Let (X, T_1) and (Y, T_2) be two Topological spaces then a mapping f : X $\rightarrow$ Y is open if and only if $f(A^\circ) \subseteq [f(A)]^\circ$ for every subset A of X.

   (b) Let (X,T_1) and (Y, T_2) be two Topological spaces then a function f : X $\rightarrow$ Y is $T_1 \rightarrow T_2$ continuous if and only if for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.

GROUP 'C'

8. (a) If f and g are two bounded and R-integrable functions in [a, b] then prove that fg is bounded and R-integrable in [a, b].

   (b) If f and g are bounded and R-integrable on [a,b] then prove that f+g is also bounded and R-integrable on [a, b] and $\int_a^b \{f(x) + g(x)\}dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.

9. (a) If a function f is continuous on [a, b] then prove that it is integrable on [a, b].

   (b) Prove that every bounded monotonic function $f:[a,b] \rightarrow R$ is R-integrable on [a, b].

GROUP 'D'

10. Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n (\log\log n)^p}$.

11. (a) Prove that the series $\sum\left(\frac{\cos n\theta}{n^2}\right)$ is convergent for all real values of θ.

    (b) Find the radius of convergence of the series $\sum \frac{n^n x^n}{n}$.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–VI (Honours)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Prove that the order of every element of a finite group is a divisor of the order of the group.
   (b) State and prove Lagrange's Theorem.

2. (a) Prove that every group is isomorphic to a group of one-one onto functions.
   (b) Sate and prove Caley's Theorem.

3. (a) Prove that the order of an element a of a group G is equal to the order of f(a).
   (b) Prove that if a group G has four elements then it must be abelian, group.

GROUP 'B'

4. (a) If f is a homomorphism of a group G into a group G'. Then prove that the Kernel K of G is a normal subgroup of G.
   (b) Define a normal subgroup of a group G. Show that every subgroup of an abelian group is normal.

5. (a) If $f(x) = x^4 + x^3 - 3x^2 - x + 2$ and $g(x) = x^4 + x^3 - x^2 + x - 2$. Then find the g.c.d. of f(x) and g(x) as polynomials over Q.
   (b) If R is a commutative ring with unity element then show that R is a field if and only if it has non-trivial ideals.

GROUP 'C'

6. (a) Show that the set of all real numbers in [0, 1] is not denumerable.
   (b) State and prove Schroder-Bernstein Theorem.

7. (a) If A_i is countably infinite set then prove that $\bigcup\limits_{i=1}^{\infty} A_i$ is countably infinite set.
   (b) Prove that $N \times N$ is countable.

8. (a) Prove that $2^{No} = c$.
   (b) State and prove Zorn's Lemma.

GROUP 'D'

9. (a) Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. satisfies laplacis equation.
   (b) If f (z) = u + iv is analytic function and $u - v = e^x(\text{Cos } y - \text{Sin } y)$ find f (z) in terms of z.

10. (a) Find the radius of convergence of the series $\frac{z}{2} + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^3 + \ldots\ldots\ldots\ldots\infty$.
    (b) Find the domain of the convergence of the series $\sum\limits_{n=1}^{\infty} \frac{1.3.5\ldots\ldots\ldots(2n-1)}{n\,!}\left(\frac{1-z}{z}\right)^n$.

11. State and prove Cauchy integral formula.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–VII (Honours)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) Prove that a sphere is a convex set.

   (b) Prove that the set of all feasible solutions of a linear programming problem constitutes a convex set.

2. Solve the L.P.P. problem by simplex method.
   Maximize $z = 4x_1 + 10x_2$. Subject to the conditions.

   $2x_1 + x_2 \le 50$
   $2x_1 + 5x_2 \le 100$
   $2x_1 + 3x_2 \le 90$
   $x_1, x_2 \ge 0.$

3. Maximize $z = 3x + 5y + 4z$. Subject to the conditions.

   $2x + 3y \le 8$
   $2y + 5z \le 10$
   $3x + 2y + 4z \le 15$
   $x, y, z \ge 0.$

GROUP 'B'

4. (a) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$.

   (b) Solve $(2xz - yz)dx + (2yz - zx)dy - (x^2 - xy + z^2)dz = 0$.

5. (a) Solve $\frac{dx}{dt} + 4x + 3y = t^2$ and $\frac{dy}{dt} + 2x + 5y = e^{2t}$.

   (b) Solve $t\frac{dx}{dt} + y = 0$ and $t\frac{dy}{dt} + x = 0$.

6. (a) Solve $r - t\cos^2 x + p\tan x = 0$ by Monge's method.

   (b) Solve $r = a^3 t$ by Monge's method.

7. (a) Solve $(p^2 + q^2)y = qz$ by Charpits method.

   (b) Solve $pxy + pq + qy - yz = 0$ by Charpit's methods.

8. (a) Solve $(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$.

   (b) Solve $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$

GROUP 'C'

9. (a) Find the attraction of a circular disc at an external point at height h.

   (b) Find the potential of a circular disc at a point distant h on the axis from the centre.

10. Find the centre of pressure of a vertical circle of radius 'a' wholly immersed in a homogeneous liquid with its centre at a depth h below the free surface.

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NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–VIII (Honours)

Annual Examination, 2022

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
General Calculator is Allowed.

1. (a) Prove that :

$$U_1 x + U_2 x^2 + U_3 x^3 + \ldots\ldots\ldots = \frac{x}{1-x} U_1 + \frac{x^2}{(1-x)^2} \Delta U_1 + \frac{x^3}{(1-x)^3} \Delta^2 U_1 + \ldots\ldots\ldots\ldots$$

(b) Show that if n is a positive integer then : $(x\Delta)^n U_x = (x + n - 1)^{(n)} \Delta^n U_x$.

2. (a) Show that if Δ operates on n, then :

$\Delta \binom{n}{x+1} = \binom{n}{x}$ and hence deduce that $\sum_{n=1}^{N} \binom{n}{x} = \binom{N+1}{x+1} - \binom{1}{x+1}$.

(b) Prove that :

$$U_x - U_{x+1} + U_{x+2} - U_{x+3} + \ldots\ldots\ldots = \frac{1}{2}\left[U_{x-\frac{1}{2}} - \frac{1}{2}\Delta^2 U_{x-\frac{3}{2}} + \frac{1.3}{2!8^2}\Delta^4{}_{x-\frac{5}{2}} + \ldots\ldots\ldots\ldots\right].$$

3. (a) If f(x) and g(x) are any functions of x then prove that :

(i) $\Delta[f(x)\, g(x)] = f(x)\Delta\, g(x) + \Delta g(x+1) f(x) \;=\; f(x+1)\Delta\, g(x) + g(x)\Delta f(x)$

(ii) $\Delta\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)\, g(x+1)}$.

(b) Express the following functions and their differences in the factorial notation.

(i) $y = x^4 - 12x^3 + 42x^2 - 30x + 9.$ (ii) $y = 2x^3 - 3x^2 + 3x - 10$

4. (a) Estimate the missing figure in the following table :

x :	1	2	3	4	5
$f(x)$:	2	5	7	X	32

(b) Find the sixth term of the series : 8 + 12 + 19 + 29 + 42 + ...

5. (a) Prove that : $\frac{\Delta^n O^m}{\lfloor n} = \frac{n\Delta^n O^{m-1}}{\lfloor n} + \frac{\Delta^{n-1} O^{m-1}}{\lfloor n-1}$.

(b) Prove that : $\Delta^n O^{n+1} = \frac{n(n+1)}{2}\Delta^n O^n$.

6. Find the maximum and minimum values of the function tabulated below.

x :	0	1	2	3	4	5
$f(x)$:	0	0.25	0	2.25	16.00	56.25

7. (a) What is the form of the function of the following table.

x :	0	1	4	5
$f(x)$:	8	11	68	123

(b) Find the polynomial of the lowest degree which assumes the values 3, 12, 15, –21. When x has the values 3, 2, 1, –1 respectively.

8. Solve the equation $2 + \log_{10}^{x} = 2e^{-x}$ by the method of iteration.

9. If f(20) = 14, f(24) = 32, f(28) = 35, f(32) = 40. Then by Gauss's forward formula show that f(25) = 33 · 49.

10. (a) Evaluate $\int_0^6 \frac{1}{1+x^2}dx$ using Simpson's $\frac{1}{3}$ rd rule.

(b) Find the solution of the difference equation $u_{x+4} - 7u_{x+1} + 12u_x = \cos x$.

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