NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–I

(Advanced Abstract Algebra)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove fundamental theorem of Galois theory.
2. State and prove Kronecker's theorem.
3. What do you mean by extension of a field. Establish the transitivity property of finite extension of a field.
4. Define Homomorphism and Kernel of homomorphism from a module M into a module N. If f is a module homomorphism then f is an isomorphism if and only if $K(f) = 0$. Prove this.
5. State and prove Jordan-Holder theorem on any group.
6. (a) Prove that the range of homomorphism of a module is a sub-module of the module.
   (b) Prove that in every principal ideal domain, each pair of elements has a greatest common divisor.
7. (a) If a and b are algebraic over a field F then prove that $a + b$, ab, ab^{-1} $(b \neq 0)$ are also algebraic over F.
   (b) Define algebraic and simple extension of a field and give an example of each one.
8. (a) Find the Galois group of the equation $x^3 - 2 = 0$ over the field Q of rational numbers.
   (b) Prove that if $K = \phi(\sqrt{2})$ where ϕ is the field of all rational numbers then ϕ is the fixed field under the group of automorphism of K.
9. (a) Construct all the composition series of Z_{60}.
   (b) Define a subnormal series of a group. Hence or otherwise form a subnormal series of the additive group of integers.
10. (a) Define a sub-module of a module M. Show that arbitrary intersection of sub-modules of a module M is a sub-module of M.
    (b) Show that a module M is the direct sum of two modules M_1 and M_2 if and only if (i) $M_1 + M_2$ and (ii) $M_1 \cap M_2 = \{0\}$ are sub modules.

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EXAMINATION PROGRAMME-2023

M.Sc. Mathematics, Part-I

Date	Papers	Time	Examination Centre
31.05.2023	Paper–I	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
02.06.2023	Paper–II	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
05.06.2023	Paper–III	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
07.06.2023	Paper–IV	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
09.06.2023	Paper–V	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
12.06.2023	Paper–VI	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
14.06.2023	Paper–VII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
16.06.2023	Paper–VIII	2.30 PM to 5.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–II
(Real Analysis)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove a necessary and sufficient condition for a function f to be R-integrable over [a, b].
   (b) If $f, g \in R(\alpha)$ on [a, b] then prove that $f + g \in R(\alpha)$ and $\int_a^b (f+g)d\alpha = \int_a^b fd\alpha + \int_a^b gd\alpha$.

2. (a) Deduce Bolzano-Weierstrass theorem from Heine-Borel theorem.
   (b) State and prove Bolzano-Weierstrass theorem and give a suitable example of it.

3. If $f : R^2 \to R$ be defined by $f(x, y) = \frac{xy(x^2 - y^2)}{x^2 + y^2}$, $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ then show that $D_{1,2}\ f(0, 0) \neq D_{2,1}\ f(0, 0)$.

4. (a) If $f \in R(\alpha)$ on [a, b] and $\int_a^b f\, d\alpha = 0$ for every f which is monotonic on [a, b] then prove that α must be constant on [a, b].
   (b) If $f \in R(\alpha)$ and α is monotonically increasing on [a, b], then show that $|f| \in R(\alpha)$ on *[a, b]* and $\left|\int_a^b f\, d\alpha\right| \leq \int_a^b |f|\, d\alpha$.

5. (a) State and prove Abel's theorem.
   (b) Find the radius on convergence of the series $\sum_{n=1}^{\infty} \frac{\lfloor n}{n^n} x^n$.

6. State and prove implicit function theorem.

7. Prove that a necessary and sufficient condition for a function f on [a, b] to be of bounded variation is that it can be written as the difference of two monotonically increasing functions on [a, b].

8. Find $\frac{\partial(y_1, y_2, y_3, ---, y_n)}{\partial(x_1, x_2, x_3, ---, x_n)}$ where

   $y_1 = x_1 (1 - x_2)$
   $y_2 = x_1 x_2 (1 - x_3)$
   $y_3 = x_1 x_2 x_3 (1 - x_4)$

   $y_{n-1} = x_1 x_2 x_3 - - - x_{n-1} (1 - x_n)$
   $y_n = x_1 x_2 x_3 - - - x_n$

9. State and prove inverse function theorem.

10. What do you mean by the Extreme values of a function in the case of a function of n-variables and find these values in the case of the function defined by $f(x, y, z) = 2xyz - 4zx - 2yz + x^2 + y^2 + z^2 - 2x - 4y + 4z$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–III
(Measure Theory)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that the class of all measurable functions is closed with respect to all algebraic operations.
   (b) If f is a measurable function then show that $|f|$ is also a measurable function.

2. (a) Show that the measure of a Denumerable set is Zero.
   (b) If A, B are L-measurable subsets of R^k then prove that $A \cup B$, $A \cap B$ are also L-measurable subsets of R^k .

3. (a) If (A_n) is sequence of L-measurable subsets of R^k such that $A_1 \supseteq A_2 \supseteq A_3 \supseteq$ .... $\supseteq A_n \supseteq A_{n+1}$ .... and $A = \bigcap_{n=1}^{\infty} A_n$ and $m(A_1) < \infty$ then show that A is L-measurable and $m(A) = \underset{n \to \infty}{lt}\ m(A_n)$.
   (b) If (S_r) is a Sequence of L-measurable subsets of R^k then show that $\bigcup_{r=1}^{\infty} S_r$ is also L-measurable.

4. Give the analytic description of Cantor's Ternary set and show that it is an uncountable set of measure Zero.

5. If (f_n) is a sequence of measurable functions then show that the class of all measurable functions is closed with respect to all analytic operations.

6. Define the Lebesque integral of a function in details. If f and g are L-integrable functions then show that $\int (f+g)d\mu = \int f\, d\mu + \int g\, d\mu$.

7. State and prove Fatou's Lemma.

8. State and prove Lebesque monotone convergence theorem.

9. (a) Examine the L-integrability of $f(x) = \left(x^2 \sin \frac{1}{x^2} \right)$ over [0, 1].
   (b) Verify bounded convergence theorem for $f_n(x) = \frac{nx}{1+n^2x^2}$ $(0 \le x \le 1)$, $n = 1, 2, 3, --$.

10. State and prove Lebesque dominated convergence theorem.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–IV
(Topology)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) Show that every metric space is a normal space.

   (b) Prove that a topological space (X, T) is normal space if and only if each neighbourhood of a closed set *F* contains the closure of some neighbourhoods of *F*.

2. (a) Prove that a topological space (X, T) is T_0 -Space if and only if $x, y \in X$ and $x \neq y \Rightarrow \{x\} \neq \{y\}$.

   (b) Define hereditary and topological properties and show that the property of a T_1-space is both hereditary and topological.

3. (a) Give an example of topological space which is a T_1-space but not a T_2-space.

   (b) Prove that a finite sub-set of T_1-space has no cluster point.

4. (a) What do you mean by a regular space. Prove that a compact Hausdorff space is regular.

   (b) Prove that every compact subspace of the real line is closed and bounded.

5. If X and Y are topological spaces, then prove that X × Y is connected *iff* X and Y are connected.

6. Prove that an arbitrary intersection of topological spaces is a topological space.

7. (a) Introduce the concept of connected and disconnected spaces and show that a topological space X is connected *iff* ϕ and X are its only subsets which are both open and closed.

   (b) Show that connectedness is not hereditary property.

8. (a) Define T_3-space and T_4-space and prove that every T_4-space is a T_3-space.

   (b) Prove that every compact subspace of a Hausdorff space is closed.

9. (a) Prove that in a Hausdorff space every convergent sequence has a unique limit.

   (b) Let (X, T) be a topological space and $A \subseteq X$. Then show that

   (i) $(Int\ A)' = \overline{A'}$, (ii) $(\overline{A})' = Int(A')$

10. (a) If (X,T) is a topological space and $A \subseteq X$, $B \subseteq X$ then show that $Int(A \cap B) = Int(A) \cap Int(B)$

    (b) Show that the open interval (0, 1) on the real line R is not compact.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-I

PAPER–V

(Linear Algebra, Lattice Theory and Boolean Algebra)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Let *V(F)* be a finite dimensional vector space and *W* is a subspace of *V*, then show that $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

2. Prove that a linear operator *E* is a projection on some subspace *iff* it is an idempotent.

3. (a) Prove that two real quadratic forms are equivalent iff they have the same rank and index.
   (b) If *f* is a linear functional on a vector space *V(K)* then show that (i) $f(0) = 0$ and $f(-x) = -f(x)$.

4. (a) Show that the relation precedes $(x \leq y)$ in a Boolean algebra *B* is a partial order relation.
   (b) If *R* is a ring and *L* is a lattice of all ideals of *R*, then prove that *L* is a modular.

5. (a) Prove that a necessary and sufficient condition for a one to one and onto mapping *f* between two lattices to be isomorphism is that f and f^{-1} are both order preserving.
   (b) Define isomorphism between two lattices. Give one example.

6. (a) Prove that a partially ordered set $(P(X), \subseteq)$ is a lattice.
   (b) If *B* is a Boolean algebra then prove that for $\forall x, y \in B$ the following are equivalent.
   (i) $x \wedge y' = 0$ (ii) $x \vee y = y$ (iii) $x' \vee y = 1$ (iv) $x \wedge y = x$

7. Convert $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$ to Jordan canonical form.

8. (a) Prove that a Boolean Algebra *B* is a complemented distributive lattice.
   (b) Prove that in Boolean Algebra, the complement of an element is unique.

9. Define a linear transformation and its null space. If *U(f)* and *V(f)* are two vector spaces and *T* is a linear transformation from *U* into *V*, then show that the kernel *T* or null space of *T* is a subspace of *U*.

10. (a) Show that the matrix, $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent of index 3.
    (b) Let {(1, 1, 1), (0, 1, 1), (0, 0, 1)} be a basis of Euclidean space R^2, then find its orthogonal basis.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VI
(Complex Analysis)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove Cauchy-Hadmard theorem for power series.

   (b) Describe the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{2^n + 1}$.

2. (a) Describe different kinds of singularities.

   (b) What is the pole of a function ? Also introduce the residue at simple pole and pole of order m.

3. Find Taylor's expansion of the function $f(z) = \frac{z}{z^2 + 9}$, around $z = 0$.

4. (a) Find the necessary and sufficient condition for analyticity of the function $f(z)$.

   (b) Show that $u = ½ \log(x^2 + y^2)$ is a harmonic function. Also find the analytic function $f(z)$ whose real part is u.

5. (a) Show that the transformation $w = \frac{5 - 4z}{4z - 2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the w–plane and hence find its centre.

   (b) State and prove the necessary and sufficient condition for the transformation $w = f(z)$ to be conformal.

6. By introducing linear transformation, derive the existence of fixed points of a Bilinear transformation.

7. Using Cauchy's integral formula evaluate $\int_C \frac{z\,dz}{(9 - z^2)(z + 1)}$, where C is the circle described anticlockwise and having equation $|z| = 2$.

8. State and prove Cauchy's theorem.

9. Evaluate the following integrals :—

   (a) $\int_0^{\infty} \frac{dx}{(1 + x^2)^2}$

   (b) $\int_0^{2\pi} \frac{d\theta}{1 + a \, Cos\theta}$ *where* $a^2 < 1$

10. State and prove Poisson's integral formula.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VII
(Theory of Differential Equations)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. State and prove Picard's-Lindelof theorem.
2. Define Lipschitz condition in a region. Show that the following function does not satisfy the Lipschitz condition in the region indicated $f(x, y) = \frac{\sin y}{x}$, $f(0, y) = 0$, $|x| \le 1$, $|y| < \infty$.
3. (a) Find an interval I containing Γ and a solution g of $y' = \frac{dy}{dx} = f(x, y)$ on I satisfying $g(\Gamma) = s$.
   (b) Compute the first three successive approximations for the solution of the equation $y' = y^2$; $y(0) = 1$.
4. (a) Determine the constants M *and* C and x for the initial value problem $y' = y$, $y(0) = 1$, $R = \{(x, y) : |x| \le 1 \text{ and } |y - 1| \le 1\}$.
   (b) Find e^A if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.
5. Prove that a necessary and sufficient condition that a solution matrix G be a fundamental matrix is that $G(x) \ne 0$ for $x \in I$.
6. Solve by matrix method the system of equations $\frac{dx_1}{dt} = 9x_1 - 8x_2$; $\frac{dx_2}{dt} = 24x_1 - 8x_2$, where $x_1(0) = 1$ and $x_2(0) = 0$.
7. (a) Find the nature of the critical point (0, 0) of the system $\frac{dx}{dt} = x + 5y$, $\frac{dy}{dt} = 3x + y$ and discuss their stability.
   (b) Explain different type of critical points for a system and give the geometrical meaning of each critical point.
8. (a) Explain the nature of critical point of a non-linear system $\frac{dx}{dt} = ax + by + \phi(x, y)$ and $\frac{dy}{dt} = cx + dy + \psi(x, y)$.
   (b) Determine the type and stability of the critical point (0, 0) of the non-linear system $\frac{dx}{dt} = Sinx - 4y$; $\frac{dy}{dt} = Sin2x - 5y$.
9. Find the Rodrigue's formula for Legendre polynomial.
10. (a) Derive an expression for the generating function for Bessel's function.
    (b) Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a +ve integer.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-I
PAPER–VIII
(Set Theory, Graph Theory, Number Theory and Differential Geometry)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) State and prove Schroder-Bernstein theorem.
   (b) Prove that $2^{N_0} = C$, where N_0 is the cardinal number of the set N and C is the cardinal number of [0, 1].
2. (a) If A and B are two countable sets then show that A × B is also countable.
   (b) Define a countable set. Prove that [0, 1] is uncountable.
3. (a) State Axiom of choice and Zermelo's postulates. Show that Axiom of choice is equivalent to Zermelo's postulates.
   (b) For any three cardinal number α, β, γ; show that (i) $\alpha^{\beta}\alpha^{\gamma} = \alpha^{\beta+\gamma}$, (ii) $(\alpha\beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$.
4. (a) If g is a connected graph with e-edges and v-vertices, then prove that $e \le 3v - 6$.
   (b) Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph.
5. (a) What do you mean by a complete graph. Show that a complete graph of n vertices is a planner if $n \le 4$.
   (b) Prove that a pseudograph is Eulerian iff it is connected and every vertex is even.
6. (a) Define isomorphism between two graphs and give two examples of isomorphic graphs.
   (b) Define the difference between a circuit and Eulerian circuit.
7. (a) Show that $(a, m_1) = 1, (a, m_2) = 1 \Leftrightarrow (a, m_1 m_2) = 1$.
   (b) Define congruency between two integers under a positive integer m. Prove that the relation $a \equiv b \pmod{m}$ defines an equivalence relation on the set of integers.
8. (a) State and prove Chinese remainder theorem.
   (b) State and prove the division algorithm of integers.
9. (a) State and prove Fermat's theorem.
   (b) If $x \equiv a \pmod 7, \equiv b \pmod{11} \equiv c \pmod{13}$ then prove or disprove that $x \equiv -286a + 364b - 77c \pmod{1001}$.
10. (a) What is a circular helix ? Find the osculating plane at the point $P(\theta)$ on the helix $x = a\, Cos\theta,\ y = a\, Sin\,\theta,\ z = c\theta$.
    (b) Prove that $\left[\vec{r}^{\,I}, \vec{r}^{\,II}, \vec{r}^{\,III}\right] = \dfrac{T}{\rho^2}$. Where $\vec{r}$ is the current point, T is torsion and ρ is the radius of curvature.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–IX
(Numerical Analysis)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
Calculator is Allowed.

1. Determine the value of the integral $\int_{4}^{5.2} \log x\, dx$ by Trapezoidal method.

2. Solve the equation,
$y_{x+3} - y_{x+2} - y_{x+1} - y_x = 0$, where $y_0 = 2,\ y_1 = -1,\ y_2 = 3$.

3. (a) Prove that $(1 + \Delta)(1 - \nabla) = 1$.

   (b) Compare Newton's method with Regula-Falsi method. Apply Newton's Raphson method to find square root of 12 to five places of decimals.

4. Define factorial notation and prove that $(x)^{(-n)} = \dfrac{1}{(x + hn)^{(n)}}$. Where h is the interval of differencing.

5. (a) Find all the real roots of the equation $x^2 + 4\,Sin\,x = 0$ correct to four places of decimals.

   (b) Obtain the missing term in the following table :—

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
f(x)	0.135	?	0.111	0.100	?	0.082	0.024

6. Form the difference equation corresponding to the family of curves $y_x = ax^2 + bx - 3$.

7. Find the formula for Quadrature for equally spaced arguments and hence derive Simpson's three-eighth rule.

8. Find the positive root of $xe^x - 1 = 0$ lying between 0 and 1 using iteration method.

9. Form Gauss's central difference table and apply it to determine e^{117} from the table :—

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

10. Compute $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at $x = 2.03$ by Newton's Backward difference formula using the following table :—

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

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<u>REVISED</u>
EXAMINATION PROGRAMME-2023
M.Sc. Mathematics, Part-II

Date	Papers	Time	Examination Centre
25.11.2023	Paper–IX	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
28.11.2023	Paper–X	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
30.11.2023	Paper–XI	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
02.12.2023	Paper–XII	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
04.12.2023	Paper–XIII	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
06.12.2023	Paper–XIV	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
08.12.2023	Paper–XV	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna
09.12.2023	Paper–XVI	10.30 AM to 1.30 PM	Nalanda Open University, 2nd Floor, Biscomaun Bhawan, Patna

NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–X
(Functional Analysis)
Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. State polarization identity and explain about it in an inner product space.

2. If $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, then prove that dual of l_p is l_q.

3. Give an example of a Banach space which is not a Hilbert space.

4. (a) If M and N are closed linear sub spaces of a Hilbert space H such that $M \perp N$ then prove that the linear sub space M + N is also closed.

   (b) Let L be a linear space over F, then show that the sum of two inner products on L is also an inner product on L.

5. Define a normed linear space and a Banach space. In a normed linear space prove that $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.

6. (a) Prove that $x_n \to x$ $w.r.t.$ $\| \;\|$ if and only if $x_n \to x'$ $w.r.t.$ $\|x\|'$.

   (b) Let X and Y be two normed linear spaces where X is finite dimensional. Then show that every linear map from X to Y is continuous.

7. State and prove Hahn-Banach theorem.

8. Sate and prove F. Riestz's theorem.

9. (a) If T is a continuous linear transformation of a Banach space X into Banach space Y, then show that T is an open mapping.

   (b) If the mapping $T \to T'$ is norm preserving mapping of $\beta(N)$ to $\beta(N')$ then prove that,

   (i) $(\alpha T_1 + \beta T_2)' = \alpha T_1' + \beta T_2'$, and

   (ii) $(T_1 T_2)' = T_2' T_1'$.

10. (a) If a Hilbert space H is separable, then show that every orthonormal set of H is countable.

    (b) If H is a Hilbert space, then show that the conjugate space H* is also a Hilbert space.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XI
(Partial Differential Equations)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Solve,
   (a) $(D^2 - DD' + D' - 1)z = Cos(x + 2y) + e^y$.
   (b) $(D - D'^2)z = Cos\ (x - 3y)$
2. Explain Charpit's method for the solution of non-linear partial differential equation of the first order.
3. Find the general solution of the partial differential equation $px(x + y) - qy(x + y) = (x - y)(2x + 2y + z)$.
4. By using the method of separation of variables solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.
5. (a) Describe Jacobi's method to solve the partial differential equation $F(x,\ y,\ z,\ p,\ q) = 0$.
   (b) Solve the partial differential equation $xyr + x^2 s - yp = x^3 e^y$.
6. Using Charpit's method solve the following partial differential equations :—
   (a) $(p^2 + q^2)y = qz$
   (b) $2zx - px^2 - 2qxy + pq = 0$
7. (a) A rod of length ℓ with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to *0°c* and are kept at that temperature. Find the temperature *u (x, t).*
   (b) Derive the Fourier equation of heat conduction.
8. Reduce $yr + (x + y)s + xt = 0$ to canonical form and hence solve it.
9. Solve the boundary value problem $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2},\ \ 0 \le x \le t,\ t > 0$ subject to the boundary conditions $\left.\begin{matrix} u\,(0,\ t) = 0,\ t > 0 \\ \frac{\partial u}{\partial x}(\ell,\ t) = 0,\ t > 0 \end{matrix}\right\}$ and the initial conditions $u\,(x,\ 0) = \begin{cases} x, & 0 \le x < \frac{\ell}{4} \\ \frac{\ell}{2} - x; & \frac{\ell}{4} \le x < \frac{\ell}{2} \\ 0; & \frac{\ell}{2} \le x < \ell \end{cases}$ and $\frac{\partial u}{\partial t}(x,\ 0) = 0,\ \ 0 \le x < \ell$.
10. (a) Show that the family of surfaces defined by $x^2 + y^2 =$ constant, is a family of equipotential surfaces in free space and hence find the law of potential.
    (b) Solve the boundary value problem $\frac{\partial u}{\partial x} = u\frac{\partial u}{\partial y}$, when $u(0,\ y) = 8e^{-3y}$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XII
(Analytical Dynamics)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Find the equation of motion of simple pendulum applying Lagrange's equation of motion.
2. Discuss the motion of a sphere when the small sphere rolls without slipping on the rough interior of a fixed vertical cylinder of greater radius.
3. (a) Prove that Lagrange's Bracket does not obey the commutative law of algebra.

   (b) Prove that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical. Find the generating function *F(q, Q)*.
4. (a) Derive Hamilton-Jacobi equation and then find Hamilton's characteristic equation.

   (b) Give the physical significance of Hamilton characteristic equation.
5. State and prove Jaicobi-Poisson theorem.
6. (a) Describe the motion of particle about revolving axes.

   (b) Using invariance of Bilinear form show that the transformation $Q = \frac{1}{p}$ and $P = p^2 q$ is canonical.
7. Derive the formula for kinetic energy in terms of generalized co-ordinates and express generalized components of momentum in terms of kinetic energy.
8. Derive Lagrange's equation of motion from Hamilton's canonical form of equations.
9. Discuss the motion of spherical pendulum deducing from Hamilton's canonical equations of motion.
10. What do you mean by Hamilton's function ? Find the differential equations for Hamilton's function.

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NALANDA OPEN UNIVERSITY

M.Sc. Mathematics, Part-II

PAPER–XIII

(Fluid Mechanics)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. Describe the motion of a fluid between rotating co-axial circular cylinders.
2. Derive the equation of motion under impulsive force.
3. Derive Euler's equation of motion in cylindrical polar co-ordinates.
4. Derive the equation of continuity in Cartesian form.
5. Prove that the fluid motion is possible when velocity at (x, y, z) is given by $u = \frac{3x^2 - r^2}{r^5}$, $v = \frac{3xy}{r^5}$, $w = \frac{3xz}{r^5}$.
6. A velocity field is given by $\vec{q} = \frac{x\vec{j} - y\vec{i}}{x^2 + y^2}$; calculate the circulation round the square having corners at (1, 0), (2, 0), (2, 1) and (1, 1). Also test for the flow of rotation.
7. (a) Show that the velocity field defined at a point P by $(1+2y-3z,\ 4-2x+5z,\ 6+3x-5y)$ represents a rigid body rotation.

   (b) Derive the rate of strain tensor of fluid in motion.
8. (a) A velocity field is given by $\vec{q} = -x\hat{i} + (y+t)\hat{j}$. Find the stream function and the stream lines for field at $t = 2$.

   (b) What do you mean by Source, Sink and Doublet. Describe them with suitable examples of each.
9. (a) The velocity $\vec{q}$ in a three dimensional flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}$. Determine the equations of streams passing through the point (1, 1, 1).

   (b) Write notes on the following :–
   (i) Velocity Potential (ii) Velocity Vector (iii) Boundary Surface
10. Derive Navier-Stokes equation of motion of viscous fluid.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XIV
(Operation Research)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.

1. Solve the following L.P.P. by using two phase simplex method
   Min $z = x_1 + x_2$
   Subject to $2x_1 + x_2 \geq 4,\ x_1 + 7x_2 \geq 7;\ x_1, x_2 \geq 0$.

2. (a) If (1, 2, 3) is a feasible solution of the set of equations $4x_1 + 2x_2 - 3x_3 = 1$; $6x_1 + 4x_2 - 5x_3 = 1$ then reduce the F.S. to B.F.S. of the set.
   (b) Solve the following L.P.P. problem by any method of your choice (except graphically)
   Max $z = 5x_1 + 7x_2$ s.t.
   $x_1 + x_2 \leq 4,\ 3x_1 + 8x_2 \leq 24,\ 10x_1 + 7x_2 \leq 35$ *and* $x_1, x_2 \geq 0$.

3. (a) Define a convex set in R^n. Let S and T be two convex sets in R^n; then show that for any scalars K_1 and K_2, $K_1 S + K_2 T$ is also a convex set in R^n.
   (b) Show that every extreme point of the convex set of feasible solution is a B.F.S. (Basic Feasible Solution).

4. Find the dual of the following L.P.P.
   Min $z = x_1 + x_2 + x_3$
   Such that $x_1 - 3x_2 + 4x_3 = 5,\ x_1 - 2x_2 \leq 3,\ 2x_2 - x_3 \geq 4;\ x_1, x_2 \geq 0$ *and* x_3 is unrestricted in sign.

5. (a) If X_0 and W_0 are feasible solutions to the primal and dual respectively then prove that $c X_0 \leq W_0 b$.
   (b) Prove that dual of the dual of a given primal is the primal itself.

6. Solve the following L.P.P. problem by simplex method.
   Minimize $z = x_1 - 3x_2 + 2x_3$
   Subject to $3x_1 - x_2 + 2x_3 \leq 7,\ -2x_1 + 4x_2 \leq 12,\ -4x_1 + 3x_2 + 8x_3 \leq 10;\ x_1, x_2, x_3 \geq 0$.

7. Solve the following assignment problem represented by the following matrix.

	I	II	III	IV	V	VI
A	9	22	58	11	19	97
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	40	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

8. Solve the following NLPP using the method of Lagrangian multipliers
   Min $z = x_1^2 + x_2^2 + x_3^2$
   Subject to constraints $4x_1 + x_2^2 + 2x_3 = 14;\ x_1, x_2, x_3 \geq 0$.

9. The pay-off matrix of a game is given below. Find the solution of the game for A and B.

		B				
		I	II	III	IV	V
A	I	-2	0	0	5	3
	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

10. Solve the following L.P.P.
    Max $z = 10x_1 + 3x_2 + 6x_3 + 5x_4$
    S.t. $x_1 + 2x_2 + x_4 \leq 6,\ 3x_1 + 2x_3 \leq 5,\ x_2 + 4x_3 + 5x_4 \leq 3$ *and* $x_1, x_2, x_3, x_4 \geq 0$
    Also, compute the limits for a_{11} and a_{23} so that the new solution remains optimal feasible solution.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XV

(Tensor Algebra, Integral Transforms, Linear Integral Equations, Operational Research Modeling)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. (a) What do you mean by symmetric and skew symmetric tensors. Prove that a symmetric tensor of rank two has at most $\frac{1}{2}N(N+1)$ different components in V_N. Where as a skew symmetric tensor of rank two has $\frac{1}{2}N(N-1)$ independent components in V_N.
   (b) State and prove quotient theorem of tensors; give an example.
2. (a) Prove that the outer product of two tensors (r, s) and (p, q) types is a tensor of $(r + s)(p + q)$ type.
   (b) Show that the co-variant derivative of a co-variant vector is a mixed tensor of rank two.
3. (a) Prove that the Laplace transform of $\frac{Sin\, at}{t}$ is $Cot^{-1}\left(\frac{s}{a}\right)$.
   (b) If $L\{F(t)\} = f(s)$ then prove that
   $L\{F^n(t)\} = s^n f(s) - s^{n-1} F(0) - s^{n-2} F'(0) \ldots\ldots\ldots\ldots - s F^{n-2}(0) - F^{n-1}(0)$.
4. (a) If a covariant tensor has components xy, $2y - z^2$, zx in rectangular co-ordinates then determine its covariant components in spherical co-ordinates.
   (b) In the matrix notation express the following transformation equations for (i) a covariant vector, (ii) a contravariant vector, (iii) a contravariant tensor of rank two assuming N = 3.
5. Solve $(D^3 - D^2 + 4D - 4)x = 68 e^t\, Sin\, 2t$. Using Laplace tranform.
6. Find the inverse Laplace transform of the following
   (i) $\frac{5s + 3}{(s - 1)(s^2 + 2s + 5)}$ (ii) $\frac{1}{s^2(s^2 + a^2)}$
7. (a) Prove that the function $u(t) + u(1 + x^2)^{-1/2}$ is a solution of the voltera integral equation.
   $$u(x) = \frac{1}{1 + x^2} - \int_0^x \frac{t}{1 + x^2}\, u(t)dt.$$
   (b) Define Firedholm integral and Voltera integral equations.
8. Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \geq a \end{cases}$ and hence evaluate
   (i) $\int_{-\infty}^{\infty} \frac{Sin\, sa\ Cos\, sx}{s} ds$ (ii) $\int_0^{\infty} \frac{Sin\, s}{s} ds$
9. Determine the deterministic model with instantaneous production. Shortage allowed.
10. Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} - \sin x \frac{dy}{dx} + e^x y = x$ with the initial conditions $y(0) = 1$ and $y'(0) = -1$.

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NALANDA OPEN UNIVERSITY
M.Sc. Mathematics, Part-II
PAPER–XVI
(Programming in 'C')

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions.
All questions carry equal marks.

1. What are logical errors and how does it differ from syntax error ? Write a programme in C to swap the value of two variables.
2. What is an Array ? How does an Array differ from an ordinary variable ? Write a program in C using array.
3. What are different types of If and else statements used in C ? Explain each of them with help of an example.
4. What are structures ? When and why are they used in C ? Give an example to explain them.
5. What is the purpose of the switch statement ? How does switch statement differ from the other statements ?
6. Describe different data types used in C programming with examples.
7. What is recursion ? Write a program in C using recursion.
8. Explain some of the looping statements with examples.
9. What is function ? Are functions required when writing a C program ?
10. Write short notes on any two of the following :—
    (i) Constant and Variables in C
    (ii) Switch statement
    (iii) Operators
    (iv) variables

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M.Sc. Mathematics, Part–II, Paper–XVI (Practical)
Counselling & Examination Programme, 2023

Practical Counselling Programme

Enrollment No.	Date	Time	Venue
210290004 to 210290355	18.12.2023 to 20.12.2023	11.00 AM to 3.00 PM	School of Computer Education & IT Nalanda Open University, Bargaon, Nalanda-803111
210290358 to 210290745	05.01.2024 to 08.01.2024	11.00 AM to 3.00 PM	
210290749 to 210291072	10.01.2024 to 12.01.2024	11.00 AM to 3.00 PM	
210291080 to 210291292, 180290294, 190290041 to 190291080	16.01.2024 to 18.01.2024	11.00 AM to 3.00 PM	
200290040 to 200291281	22.01.2024 to 24.01.2024	11.00 AM to 3.00 PM	

Practical Examination Programme

Enrollment No.	Date	Time	Venue
210290004 to 210290355	21.12.2023	12.00 Noon to 2.00 PM	School of Computer Education & IT Nalanda Open University, Bargaon, Nalanda-803111
210290358 to 210290745	09.01.2024	12.00 Noon to 2.00 PM	
210290749 to 210291072	13.01.2024	12.00 Noon to 2.00 PM	
210291080 to 210291292, 180290294, 190290041 to 190291080	19.01.2024	12.00 Noon to 2.00 PM	
200290040 to 200291281	25.01.2024	12.00 Noon to 2.00 PM	

सभी सम्बन्धितों (विद्यार्थियों) को हिदायत दी जाती है कि वे अपना Admit Card अवश्य लेकर आयें तथा निर्धारित तिथि एवं समय पर उपस्थित होवें अन्यथा परामर्श कक्षाओं एवं परीक्षाओं से वंचित हो सकते हैं । परामर्श कक्षा में उपस्थित नहीं होने पर प्रायोगिक परीक्षा में अनुत्तीर्ण हो सकते हैं, क्योंकि प्रायोगिक परीक्षा से सम्बन्धित विशेष मार्गदर्शन परामर्श कक्षा में हीं प्राप्त किया जा सकता है ।

विद्यार्थियों को यह भी सूचित किया जाता है कि नालन्दा खुला विश्वविद्यालय, बड़गाँव, नालन्दा–803111 में छात्र–छात्राओं के ठहरने के लिए अलग–अलग हॉस्टल एवं खाने के लिए कैंटीन की सुविधा उपलब्ध है । हॉस्टल (100/– रुपये प्रति बेड प्रतिदिन) एवं कैन्टीन का उपयोग इच्छुक विद्यार्थियों द्वारा किया जा सकता है, जिसका व्यय निर्धारित है ।

आई०टी० समन्वयक

इस कार्यक्रम में किसी भी परिस्थिति में परिवर्तन नहीं होगा ।