NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-I

PAPER–I (Honours)

(Set Theory, Matrices, Abstract Algebra, Theory of Equations and Trigonometry)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. State and prove fundamental theorem of equivalence relation.
2. What do you mean by a partial order relation and total order relation and well ordered set. Give one example of each.
3. If A, B, C, D are any three non-empty sets then prove that
   (a) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D) \cup (A \times D) \cup (B \times C)$
   (b) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
4. (a) Define an equivalence relation and equivalence classes of sets giving one example of each.
   (b) If $f : x \to y$ and $A \subseteq y$, $B \subseteq y$ then show that
   $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ *and* $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
5. Define a Lattice, Complete Lattice and set an example of Lattice which is not a complete Lattice.

GROUP 'B'

6. (a) H_1, H_2 are subgroups of a group G then show that $H_1 \cap H_2$ is also a subgroup of G.
   (b) Prove that a group G is abelian if $b^{-1}a^{-1}ba = e \;\forall\; a, b \in G$ and e is the identify element of G.
7. (a) Prove that the order of every element of a finite group is a divisor of the order of the group.
   (b) Prove that if a group G has four elements then it must be abelian.
8. (a) Prove that G = {0, 1, 2, 3, 4, 5} is a finite abelian group of order 6 with respect to addition modulo 6.
   (b) Define a group and show that the four fourth roots of unity, namely 1, -1, i, -i form a group with respect to multiplication.

GROUP 'C'

9. Find the eigen values and eigen vectors of the matrix : $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.
10. (a) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix}$.
    (b) If A and B are any two non-singular matrices of the same order then prove that $(AB)^{-1} = B^{-1}A^{-1}$.
11. (a) Solve the following system of linear equations by matrix method.
    $$\left.\begin{array}{l} x + y + z = 6 \\ 2x + y - 3z = -5 \\ 3x - 2y + z = 2 \end{array}\right\}.$$
    (b) If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$; Then find the value of $A^2 - 4A + 3I$.
12. (a) Find the condition so that the equation $x^4 - px^3 - qx^2 + rx + s = 0$ may have its roots in arithmetical progression.
    (b) State and prove De-Moiver's theorem.

• • •

NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-I
PAPER–II (Honours)
(Differential Calculus, Integral Calculus and Analytical Geometry of Three Dimensions)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. Evaluate :— (a) $\underset{x\to 0}{Limit}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. (b) $\underset{x\to 0}{Limit}\ \frac{xe^x - \log(1+x)}{x^2}$

2. (a) If the normal at any point to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with x-axis then show that its equation is $y\,Cos\phi - x\,Sin\phi = a\,Cos2\phi$.

   (b) If $u = \log\left(x^2 + y^2 + z^2 - 3xyz\right)$ then show that : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{3}{(x^2+y^2+z^2)^2}$.

3. (a) Find the Lagrange's form of remainder after n terms in the expansion of $e^{ax}\cos bx$ in powers of x.

   (b) State and prove Taylor's theorem.

4. (a) If $y = \left(x^2 - 1\right)^n$ then prove that, $\left(x^2 - 1\right)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

   (b) If $y = e^{a\,Sin^{-1}x}$ then prove that, $\left(1 - x^2\right)y_{n+2} - (2n+1)xy_{n+1} - \left(n^2 + a^2\right)y_n = 0$.

5. (a) Find the asymptotes to the curve $\left(x^2 + y^2\right)(x + 2y - 2) = x + 9y + 2$.

   (b) Prove that the radius of curvature for the pedal curve $p = f(r)$ is given by $\rho = r\frac{dr}{dp}$.

GROUP 'B'

6. (a) Obtain the reduction formula for $\int Cos^m x\,Sin\,nx\,dx$.

   (b) Evaluate $\underset{n\to\infty}{Limit}\left(\sum_{n=1}^{n-1}\frac{1}{n}\sqrt{\frac{n+r}{n-r}}\right)$

7. Evaluate any ***Two*** of the following :—

   (a) $\int \cos ec^3 x\ dx$ (b) $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$ (c) $\int \frac{x^2}{x^4+1}dx$

8. Evaluate any ***Two*** of the following :—

   (a) $\int_0^{\pi/4}\log(1+\tan\theta)d\theta$ (b) $\int_0^{\pi/2}\frac{Sin^2 x}{Sinx + Cosx}dx$ (c) $\int_0^{\pi/2}\log(Sinx)dx$

9. Find the volume of the solid formed by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x-axis.

10. Find the area of the loop $y^2 = x(x-1)^2$.

GROUP 'C'

11. (a) Find the polar equation of the tangent at any point of it to the conic $\frac{\ell}{r} = 1 + e\,Cos\theta$.

    (b) Find the polar equation of the conic in the form $\frac{\ell}{r} = 1 + e\,Cos\theta$.

12. (a) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y - 15 = 0$.

    (b) If the tangent to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts on the co- ordinate axis a, b, c, respectively then show that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{r^2}$.

• • •

NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-I, PAPER–I (Subsidiary)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer any **Eight** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. What do you mean by an Equivalence relation ? Give two examples of it.

2. Let $f: X \to Y,\ A \subseteq Y,\ B \subseteq Y$ then show that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.

3. If A, B, C are any three non-empty sets then prove that

   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

GROUP 'B'

4. Let G = {1, w, w^2} where w is an imaginary cube root of unity then prove that G is a group with respect to multiplication as operation.

5. What do you mean by an abelian group ? If a group G has four elements then prove that it must be abelian group.

6. For a finite group G, prove that the order of every element of G is finite and less than or equal to the order of the group G.

7. Let f be a homomorphism of a group G into a group G' then prove that.

   (i) f(e) = e' where e is the identity of G and e' is the identify element of G'.

   (ii) $f(a^{-1}) = \{f(a)\}^{-1}\ \forall\ a \in G$.

   (iii) If the order of $a \in G$ is finite then the order of f(a) is the divisor of the order of a.

8. Let f be a homomorphism of a group G into a group G' with Kernel, K = {x $\in$ G : f(x) = e') where e' is the identity element of G', then show that K is a normal sub group of G.

GROUP 'C'

9. If $\tan(x + iy) = u + iv$ then prove that $u^2 + v^2 + 2u\,Cot\,2x = 1$.

10. State and prove De-Moivre's theorem.

11. Decompose $\log(\alpha + i\beta)$ into real and imaginary parts.

GROUP 'D'

12. Test the convergence of the series whose n^{th} term is $\left(\sqrt{n^2+1} - \sqrt{n^2-1}\right)$.

13. (a) Show that the sequence (a_n) where $a_n = \sqrt{n^2 + 4n} - n$ is convergent.

    (b) State and prove Cauchy general principle of convergence of a real sequence.

GROUP 'E'

14. (a) If $f(x, y) = x\,Cosy + y\,Cosx$ then prove that : $\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial^2 f}{\partial y\,\partial x}$.

    (b) State and prove Euler's theorem on Homogeneous functions of two variables.

15. Deduce the polar equation of the conic in the form $\frac{\ell}{r} = 1 + e\cos\theta$.

16. Prove that : $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]^2$.

17. Prove that : $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$.

● ● ●

NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-II
PAPER–III (Honours)
Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. (a) State and prove theorem of greatest lower bound.
   (b) Show that any non-empty open set is a union of open intervals.

2. (a) Prove that between two distinct real numbers there lie infinity of irrationals and rationals.
   (b) Define a closed set. Prove that the intersection of any number of closed sets is closed.

3. (a) State and prove fundamental theorem of classical analysis.
   (b) State and prove theorem of least upper bound.

GROUP 'B'

4. (a) Let $x_1 = 1, x_2 = \sqrt{2 + x_1}, x_3 = \sqrt{2 + x_2}, \ldots\ldots\ldots, x_{n+1} = \sqrt{2 + x_n}$. Show that the sequence (x_n) is convergent and the limit converges to 2.
   (b) Show that the sequence (a_n) defined by $a_1 = \sqrt{7}$, $a_{n+1} = \sqrt{7 + a_n}$ converges to a positive root of the equation $x^2 - x - 7 = 0$.

5. (a) Show that a bounded monotonic increasing sequence tends to its least upper bound.
   (b) Define a convergent sequence and show that it is bounded.

6. (a) Test the convergence of the series $\sum \frac{(n+1)(n+2)}{(n+3)(n+4)}$.
   (b) State and prove Raabe's test.

7. (a) Test the convergence of the series whose n^{th} term is $\sqrt{n^2+1} - \sqrt{n^2-1}$.
   (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{1+n^2}, \forall\, x > 0$.

8. (a) State and prove Cauchy's n^{th} root test for convergence of an infinite series.
   (b) Test the convergence of the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots\ldots\ldots\ldots \infty$.

GROUP 'C'

9. (a) Prove that any two bases of a finite dimensional vector space have the same number of elements.
   (b) Let V be a vector space and W_1, W_2 are finite dimensional subspaces of V. Then show that $W_1 + W_2$ is finite dimensional and

   dim . W_1 + dim . W_2 = dim. ($W_1 \cap W_2$) + dim. ($W_1 + W_2$).

10. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$.
    (b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$.

• • •

NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-II

PAPER–IV (Honours)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. Solve any **Two** of the following differential equations :—
   (a) $(px - y)(x - py) = 2p$
   (b) $(x - a)p^2 + (x - y)p - y = 0$
   (c) $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6 = 0$

2. (a) Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$ by using method of change of variables.
   (b) Solve $\frac{d^2y}{dx^2} + a^2y = Sec\, ax$ by using variation of parameters.

3. (a) Prove that the system of confocal conic $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal.
   (b) Find the orthogonal Trajectory of the family of Cardoids $r = a(1 + Cos\,\theta)$.

GROUP 'B'

4. (a) Prove that $\frac{d}{dt}(\vec{u}\cdot\vec{v}) = \vec{u}\cdot\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\cdot\vec{v}$.
   (b) Prove that $\frac{d}{dt}(\vec{u}\times\vec{v}) = \vec{u}\times\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\times\vec{v}$.

5. (a) Prove that : $\nabla\cdot(\vec{u}\times\vec{v}) = \vec{v}\cdot(\nabla\times\vec{u}) - \vec{u}\cdot(\nabla\times\vec{v})$
   (b) Prove that $\nabla\times(\vec{u}\pm\vec{v}) = \nabla\times\vec{u} \pm \nabla\times\vec{v}$.

6. (a) Prove that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\,\vec{b} - (\vec{a}\cdot\vec{b})\,\vec{c}$.
   (b) Prove that $\left[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$.

GROUP 'C'

7. State and prove the necessary and sufficient condition for equilibrium of a system of co-planar forces; also find the equation of line of action of the resultant.

8. State and prove the necessary and sufficient condition of the principle of virtual work.

9. Define simple Harmonic motion. If in a simple harmonic motion u, v, w be the velocities at distances a, b, c from a fixed point on the straight line which is not the centre of the force. Show that the periodic time T is given by the equation:

$$4\pi^2(a-b)\,(b-c)\,(c-a) = T\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$

10. Derive the tangential and normal velocities and accelerations in polar co-ordinates.

● ● ●

NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-II
PAPER–II (Subsidiary)
Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Eight** questions in all, selecting at least one question from each group. All questions carry equal marks.*

GROUP-A

1. Evaluate any two of the following integrals :–

   (a) $\int \frac{dx}{\sin x(3+2\cos x)}$ (b) $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ (c) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$

2. Find the reduction formula for :–

   (a) $\int Sin^m x\ Cos\, nx\, dx$ (b) $\int_0^{\pi/2} Sin^m x\ Cos^n x\, dx$

3. Evaluate any two of the following :–

   (a) $\int_0^{\pi/2} \log(\tan x) dx$ (b) $\int_0^{\pi/2} \frac{Sin^2 x}{Sinx + Cosx} dx$ (c) $\int_0^{\infty} \frac{\log(1+x^2)}{(1+x^2)} dx$

4. (a) Evaluate $\underset{n\to\infty}{lt} \frac{[(n+1)(n+2)(n+3)\ \ldots\ldots\ldots\ldots(n+n)]}{n}$

   (b) Evaluate $\underset{n\to\infty}{lt} \left[\frac{1^2}{1^3+n^3} + \frac{2^2}{2^3+n^3} + \ldots\ldots\ldots\ldots + \frac{n^2}{n^3+n^3}\right]$.

5. Find the perimeter of the loop of the curve $9ay^2 = (x - 2a)(x - 5a)^2$.
6. Find the volume of revolution of the loop of the curve $y^2(a + x) = x^2(a - x)$ about the x-axis.
7. Find the area between the curve $y^2(a + x) = (a - x)^2$ and its asymptote.
8. Solve the following differential equations :–

   (a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{2x}$ (b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4y = x^2$.

9. Solve :– (a) $y = px - x^4p^2$. (b) $y = 2px + p^2$

GROUP 'B'

10. (a) Find the equation of the sphere which passes through the point (α, β, γ) and the circle $x^2 + y^2 + z^2 = a^2$, $z = 0$.

    (b) Find the equation of the right circular cylinder whose axis is given by $\frac{x}{1} = \frac{y}{0} = \frac{z}{2}$ and radius $\sqrt{7}$.

11. (a) Prove that the intersection of a finite number of convex sets is a convex set.

    (b) Define a convex set and a hyper plane and prove that a hyper plane is a convex set.

12. Find the volume of the Tetrahedron, the co-ordinates of whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4).

GROUP 'C'

13. Deduce the general conditions for equilibrium of a system of co-planar forces.
14. (a) Analyze the motion of a body under inverse square law.

    (b) State and establish the principle of energy.

15. State and prove principle of virtual work.
16. What do you mean by Simple Harmonic Motion, derive an expression for time period.

• • •

NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–V (Honours)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. Show that every metric space is T_2-space.

2. If $1 < p < \infty$, $1 < q < \infty$ such that $\frac{1}{p}+\frac{1}{q}=1$ and a, b are real numbers such that $a > 0$, $b > 0$ then prove that ab $\le \frac{a^p}{p}+\frac{b^q}{q}$.

3. (a) Let (X, d) be a metric space. Show that a function d* : X × X → R difined by d* = $\frac{d(x,y)}{1+d(x,y)}$ is also a metric for X.

   (b) Prove that in a metric space (X, d) each open sphere is an open set.

4. State and prove Minkowskys inequality.

5. Prove that every metric space is first countable.

GROUP 'B'

6. Let (X, T) is a topological space and A and B are subsets of X. If $\overline{A}$ denotes the closure of A then show that :

   (a) $(\overline{A \cap B}) = \overline{A} \cap \overline{B}$ (b) $(\overline{A \cup B}) = \overline{A} \cup \overline{B}$ (c) $\overline{\overline{A}} = A$

7. What do you mean by a Hausdorff space, Show that every discrete topological space is a Hausdorff space.

GROUP 'C'

8. State and prove Darboux theorem.

9. State and prove necessary and sufficient condition for R-integrability of a bounded function f over [a, b].

10. Prove that if a bounded function f is R-integrable over [a, b] and M and m are bounds of f then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$ *if* $b \ge a$.

GROUP 'D'

11. Discuss the convergence of the following series :

    (a) $1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\frac{1}{5^p}+\frac{1}{6^p}+ \ldots\ldots\ldots\ldots\infty.$

    (b) $\sum_{n=2}^{\infty} \frac{1}{n \log n\, (\log\log n)^p}$

12. Show that the sum of the series $1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\frac{1}{5}-\frac{1}{10}-\frac{1}{12}+ \ldots\ldots$ is half the sum of the series $1-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}+\frac{1}{7}\ldots\ldots\ldots$

• • •

NALANDA OPEN UNIVERSITY

B.Sc. Mathematics, Part-III

PAPER–VI (Honours)

Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. Show that any ring can be embedded in a ring with unity.

2. Define a ring homomorphism. If $f: R \to R'$ be a homomorphism of a ring R onto a ring R' then show that f is a homomorphism iff Kernel of $f = \{0\}$.

3. Define the principal ideal ring and show that the ring of integers is a principal ideal ring.

4. Show that the union of two ideals is again an ideal.

5. Prove that the set of all polynomials in $Z[x]$ with constant term O is prime ideal in $Z[x]$.

6. (a) If G is a group, then for every element $g \in G$, prove that $C_o(g)$ is a Subgroup of G.
   (b) Define an automorphism of a group G. Let $x \in G$, then prove that the function f defined by $f(g) = x^{-1}gx$ for $g \in G$ is an automorphism of G.

GROUP 'B'

7. State and prove Cantor's Theorem.

8. (a) If X be any non-empty set then show that card $(P(x))$ is 2 where $P(x)$ is the power set of X.
   (b) Introduce the concept of order types and construct the product of two order types.

9. (a) For cardinal numbers α, β, γ prove that
   (i) $\alpha^{\beta} \cdot \alpha^{\gamma} = \alpha^{\beta+\gamma}$ (ii) $(\alpha \cdot \beta)^{\gamma} = \alpha^{\gamma}\beta^{\gamma}$ (iii) $(\alpha^{\beta})^{\gamma} = \alpha^{\beta\gamma}$
   (b) Prove that 2^{No} = c, where symbols have their usual meaning.

GROUP 'C'

10. Obtain the necessary and sufficient condition for differentiability of a complex valued function.

11. State and prove Cauchy integral formula.

12. (a) Evaluate $\int_C \frac{e^{2z}}{(z+1)^2} dz$. Where C is the circle $|z| = 3$.
    (b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann differential equations are satisfied.

• • •

NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-III
PAPER–VII (Honours)
Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

*Answer **Five** questions in all, selecting at least one question from each group.*
All questions carry equal marks.

GROUP 'A'

1. Use simplex method to solve :
   Maximize : $z = 3x_1 + 9x_2$
   Subject to $x_1 + 4x_2 \le 8, \quad x_1 + 2x_2 \le 4$ and $x_1 \ge 0, x_2 \ge 0$.

2. (a) Define convex combination of vectors in R^n. Prove that the set of convex combinations of a finite number of linearly independent vectors $v_1, v_2, v_3, ..., v_n$ is a convex set.
   (b) Prove that the set of all feasible solutions of a linear programming problem constitutes a convex set.

3. (a) Prove that every hyperplane is convex.
   (b) Define a convex set, the subset of R^n and show that the finite intersection of convex sets is a convex set.

GROUP 'B'

4. Solve by using Charpit's method $\left(p^2 + q^2\right)x = pz$.

5. Use Monge's method to find the complete solution of the equation
   $2x^2r - 5xys + 2y^2t + 2(px + qy) = 0$

6. Solve :
   (a) $(y + z)\, p + (z + x)q = x + y$
   (b) $pz - qz = z^2\,(x + y)^2$

7. (a) Solve $\dfrac{dx}{dt} + 4x + 3y = t$ and $\dfrac{dy}{dt} + 2x + 5y = e^t$.
   (b) Solve $\dfrac{dx}{x^2 - y^2 - z^2} = \dfrac{dy}{2xy} = \dfrac{dz}{2xz}$.

8. Test for integrability and hence solve the equation
   $\left(y^2 + yz\right)dx + \left(z^2 + zx\right)dy + \left(y^2 - xy\right)dz = 0$

GROUP 'C'

9. State and prove Laplace theorem in cartesian form.

10. Find the attraction of a uniform sphere at an external point of it.

• • •

NALANDA OPEN UNIVERSITY
B.Sc. Mathematics, Part-III
PAPER–VIII (Honours)
Annual Examination, 2023

Time : 3 Hours. **Full Marks : 80**

Answer any Five Questions. All questions carry equal marks.
(General Calculator is Allowed)

1. (a) Use Weddle's rule to evaluate $\int_0^{10} \frac{1}{1+x} dx$.
   (b) Derive Simpson's $\frac{3}{8}$th rule for numerical integration.
2. Derive Newton-Gregory formula for backward interpolation.
3. Applying analytic method for finding roots of an equation based on Rolle's theorem and demonstrate on $3x - \sqrt{1+\sin x} = 0$.
4. By using synthetic division solve
   $f(x) = x^3 - x^2 - (1.001)x + 0.9999 = 0$ in the neighbourhood of $x = 1$.
5. (a) Apply Runge-Kutta mehtod for the solution of first order differential equation.
   (b) Describe Picard's method of successive approximation.
6. (a) Evaluate $\Delta^3 (1-x)(1-2x)(1-3x)$ and $\Delta^n \left(e^{ax+b}\right)$ where a and b are constants.
   (b) Explain the meaning of the operators E and Δ. Show that E and Δ are commutative with respect to variables.
7. Use Gauss-Jordan method to solve the system of equations
   $x_1 + 2x_2 + x_3 = 8,\ 2x_1 + 3x_2 + 4x_3 = 20$ and $4x_1 + 3x_2 + 3x_3 = 16$
   taking initial conditions $x_1 = 0,\ x_2 = 0,\ x_3 = 0$.
8. (a) Solve the following system of equations

$$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = 1$$

$$\frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 = 0$$

$$\frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 = 0$$

   (b) Explain Gauss's method of elimination for the solution of a system of m equations in m variables.
9. (a) Solve difference equation $\bigcup_{x+1} = 2^x \cup_x$.
   (b) Derive Trapezoidal and Simpson's one third rule to numerical integration.
10. (a) Describe Milne corrector formula.
    (b) State and prove Adam's predictor formula.

• • •